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Dimensional Analysis in DERIVE and TI-92

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Abstract
The report presented is the continuation of one of our reports at the 2nd International DERIVE and TI-92 Conference [1]. Dimensional Analysis is a very powerful and rather simple method. It is especially helpful in the cases when the rigorous solution from the first principles is difficult to found. Due to its simplicity it can be studied at all levels of complexity from secondary school to university and research. The report consists of 5 parts: history, dimension check of equations, dimensional approach in physics equation derivation, equations transformation to the dimensionless form (including self-similarity check), and a short discussion of fruitful combination of dimensional analysis followed by the numerical one.

1. History background.

One of the first significant fruitful applications of the dimensional analysis was reported by O. Reynolds in 1883 [2], but the wide use of dimensional analysis in science started in twenties of the 20th century after the famous P.W.Bridgmans' lectures in Princeton University in 1920 (printed in 1922) [3]. A thick 402 page Russian book, "The Foundations of Qualitative Phys-Math Analysis" devoted to dimensional analysis by Nikolai A. Morozov [4], was written in late eighteen nineties and published in 1908. This book is not as well known as [2,3]. The author builds a table of dimensional expressions and it is used for different physics problems. The main difference with the modern tables is in numerical factors of the dimensional expressions that gave the author the opportunity to derive exact expressions, but not to within a dimensionless factor as it is popular today. This feature of Morozovs' method is of great interest and will be studied in our future papers. The problem is that he used not only a dimensional table, but also additional physical considerations, specific to each problem considerations, difficult to algorithmize and program. That is why our present paper deals with several examples from his book at the regular modern level.

2. Dimension Check in Equations

Due to its simplicity the foundations of the dimensional analysis can be introduced in the secondary school. Equations simplification using its variables dimension and units conversion is well supported by the TI-89/92+, but in DERIVE - not. So, we wrote the MTH utility file with the function EVAL(expr, var_val) that evaluates expression and its accuracy using variables values (& accuracy), and checks the expression and variables consistency [1]. This function can be useful from school to university students teaching. The corresponding TI-92 program with the similar

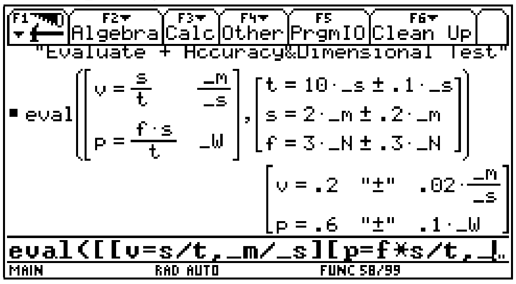

Fig.1. Substitution with the Accuracy and Dimensional Test in TI-92

syntax is also written (Fig.1).

3. Dimensional Approach in Physics Equations Derivation

5 of 7 examples of this part is from [4, pp.243-269]. In order to reduce bulky calculation and conclusions a simple DERIVE function was written. It helped us to reduce these 5 problems solutions derivation from 19 pages to 10 lines. For instance, when we search for the dependence of the Mathematical Pendulum Period t (measured in seconds [s]) on its length l [m] and acceleration of gravity g [m/s^2] the function DIM_ANAL() returns the equation to within a numerical factor 2π.

$$\mathrm{DIM\_ANALS}\begin{bmatrix} t & s \\ l & m \\ g & \dfrac{m}{s^2} \end{bmatrix} = \left[t = \frac{\sqrt{l}}{\sqrt{g}}\right]$$

The same function applied to: acceleration (a) & mass (m); velocity (v) & mass (m); acceleration of gravity (g), mass, and height (h); velocity (v) & g; v & the radius of curvature (r); return force (f); kinetic energy (Ek); potential energy (Ep); shell distance (dist); and rotation acceleration (ar) respectively:

```
F = a·mass, Ek = mass·v^2, Ep = g·h·mass, dist =v^2/g, ar = v^2/r
```

All above examples illustrate the power of dimensional analysis. The below one will show its limitations. The attempt to find expression for the uniformly accelerated rectilinear motion `S = v*t + a*t^2/2` result in vector of 5 expressions:

$$\left[s\_ = \frac{v^2}{a},\ s\_ = \frac{\sqrt{t}\cdot v^{1.5}}{\sqrt{a}},\ s\_ = t\cdot v,\ s\_ = \sqrt{a}\cdot t^{1.5}\cdot\sqrt{v},\ s\_ = a\cdot t^2\right]$$

The 3d & 5th elements are valid, the 1st is valid for rotation acceleration but not the problem under consideration, and the 2d & 4th - invalid. The problem of the valid elements selection is not trivial. The Simplicity Concept can help in some cases. The restriction to integer parameters' powers result in 3 valid expressions:

```
bound := [-2, 2, 1]
```

$$\left[s\_ = \frac{v^2}{a},\ s\_ = t\cdot v,\ s\_ = a\cdot t^2\right]$$

The appropriate function for TI-89/92 calculators with the similar name, functionality, and syntax was written (Fig.2).

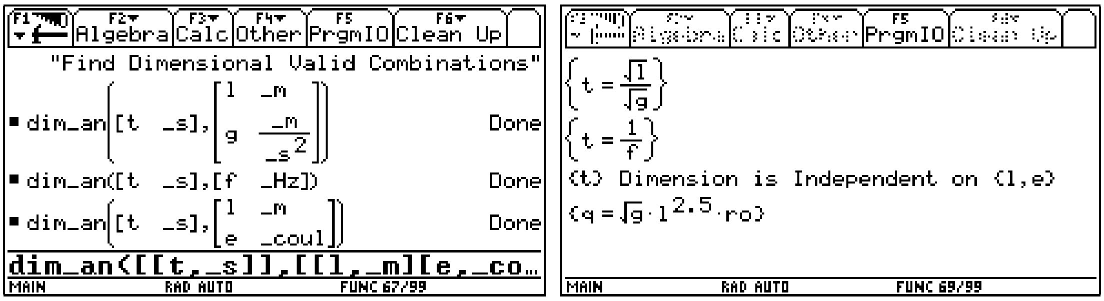

Fig.2. Dimensional Valid Expressions Derivation in TI-92 (Input -left, output -right)

4. Equations Transformation to the Dimensionless Form

Bulky transformations needed for the problem dimension reduction and self-similarity test can be done in CAS. The appropriate functions in DERIVE and TI-92+ find appropriate variables substitutions and return it with the transformed to the form dependent on new variables.

The problem of the water flow rate (Q[kg/s]) of the waterfall illustrates the number of independent variables reduction by transformation to the dimensionless form. The width of the waterfall is b[m], and the water depth at the top of the waterfall is h[m]. As the water falls due to the force of gravity, the acceleration of gravity (g[m/s^2]) must be taken into account. The process is also depend on the density of water ρ[kg/m^3]. So, the problem is defined by 5 defining parameters: Q , g, h, ρ, and b i.e. Q=f(ρ,g,h,b). First, we have to find all dimensionally independent complete sets of independent parameters. (Every defining parameter must be linear dependent on the complete independent set.)

```
bound := [-3, 3, 0.5]
```

$$\mathrm{DIM\_INDEPENDENT\_SETS}\begin{bmatrix} \rho & \dfrac{kg}{m^3} \\ g & \dfrac{m}{s^2} \\ h & m \\ b & m \end{bmatrix} = \left[\begin{bmatrix} \rho & \dfrac{kg}{m^3} \\ g & \dfrac{m}{s^2} \\ b & m \end{bmatrix}, \begin{bmatrix} \rho & \dfrac{kg}{m^3} \\ g & \dfrac{m}{s^2} \\ h & m \end{bmatrix}\right]$$

The next step is to choose one of the sets using physics problem considerations, apply π-theorem, and get a dimensionless expression:

$$\text{Pi\_Theorem}\left(\begin{bmatrix} Q & \frac{kg}{s} \\ \rho & \frac{kg}{m^3} \\ g & \frac{m}{s^2} \\ h & m \\ b & m \end{bmatrix}, \begin{bmatrix} \rho & \frac{kg}{m^3} \\ g & \frac{m}{s^2} \\ h & m \end{bmatrix}\right) = \left(\frac{Q}{\sqrt{g}\cdot\rho\cdot h^{5/2}} = f\_\left(\left[\frac{b}{h}\right]\right)\right)$$

The derived dimensionless expression depend on 1 dimensionless independent variable b/h instead of 4 (ρ,g,h,b) in the initial expression. It is evident that the univariate expression Q_=f_(b/h) study (experimental and/or numerical) is several orders more simple than for the multivariate one.

The below function result (R) is better for the case of known equations (EQ) non-dimensionalization by simplification of: `SUBST(EQ, LHS(R), RHS(R))`

$$\text{PI\_THEOREM}\left(\begin{bmatrix} Q & \frac{kg}{s} \\ \rho & \frac{kg}{m^3} \\ g & \frac{m}{s^2} \\ h & m \\ b & m \end{bmatrix}, \begin{bmatrix} \rho & \frac{kg}{m^3} \\ g & \frac{m}{s^2} \\ h & m \end{bmatrix}\right) = \begin{bmatrix} \rho & 1 \\ g & 1 \\ h & 1 \\ Q & \frac{Q}{\sqrt{g}\cdot\rho\cdot h^{5/2}} \\ b & \frac{b}{h} \end{bmatrix}$$

Note that Pi_Theorem() function returns constant in appropriate cases.

$$\text{Pi\_Theorem}\left(\begin{bmatrix} t & s \\ l & m \\ g & \frac{m}{s^2} \end{bmatrix}, \begin{bmatrix} l & m \\ g & \frac{m}{s^2} \end{bmatrix}\right) = \left(\frac{\sqrt{g}\cdot t}{\sqrt{l}} = \text{const}\right)$$

Completeness of the parameters set with the respect to another set, dimensional independency, and self-similarity [5,6] can be also tested.

5. The Role of Dimensional and Numerical Analysis in Equations and Solutions Derivation

The two step procedure of the equation derivation can be used. At the first step the dimensional analysis is used to derive terms and dimensionless factors of some function. The whole structure of the desired equation and its factors is determined at the second step using numerical analysis of the definite case of the problem or experimental one.

The one of the ways to solve a mathematical problem can be a solution of the corresponding physical problem. It is the inversion of the regular method widely used in physics: build a physical model, convert it to the mathematical one, solve and analyze math problem, and interpret the results using physical notions. In the proposed method the math problem is replaced by the corresponding physical or some other natural science model, this model is analyzed with the dimensional + numerical analysis and the result is applied to and verified for the math problem. Such approach seems rather strange but one must note that it has a close analogy with the analog computer - low accuracy super-fast competitive to the digital computer in the past and the only possible one for some real-time applications nowadays.

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