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Derive 5: The Easiest ... Just Got Better!

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1. Introduction

Engineering students at the École de Technologie Supérieure have to buy a TI-89 or TI-92 Plus and are allowed to use it in the classroom and during the tests. But that does not mean the end of *DERIVE*! And with version 5, we are able to go very deep into multivariable calculus. We will give examples of using *Derive* 5 and will try to show why, in connection with a TI calculator, *DERIVE* continues to be a perfect choice in order to teach mathematics at many levels.

2. This system is a trusted mathematical assistant

2a) Example : can your system do this ?

Everyone knows that partial fraction expansion is something that a CAS can do. From the beginning this has been possible and easy with *Derive* (and so, the following example is not a new feature of *Derive 5*). Well, let's check with some non obvious rational functions. Suppose we consider the third degree polynomial $3x^3 - 18x^2 + 33x - 19$. A graph shows three real roots : *Derive* gives the solution in a very sophisticated way. Now suppose you want to evaluate exactly the definite integral

$$\int_2^3 \frac{dx}{3x^3 - 18x^2 + 33x - 19}$$

(choosing the numbers 2 and 3 for the integration limits avoids an improper integral). Your system must be able to perform partial fraction expansion ! ! !

$$\text{SOLUTIONS}(3 \cdot x^3 - 18 \cdot x^2 + 33 \cdot x - 19 = 0,\ x)$$

$$\left[2 - \frac{2 \cdot \sqrt{3} \cdot \text{SIN}\left(\frac{\pi}{9}\right)}{3},\ 2 - \frac{2 \cdot \sqrt{3} \cdot \text{SIN}\left(\frac{2 \cdot \pi}{9}\right)}{3},\ \frac{2 \cdot \sqrt{3} \cdot \text{COS}\left(\frac{\pi}{18}\right)}{3} + 2\right]$$

$$\int \frac{1}{3\cdot x^3 - 18\cdot x^2 + 33\cdot x - 19}\,dx$$

$$\frac{\mathrm{LN}\left(2\cdot \mathrm{COS}\left(\frac{\pi}{18}\right) - \sqrt{3}\cdot x + 2\cdot\sqrt{3}\right)}{4\cdot\left(\mathrm{COS}\left(\frac{\pi}{9}\right) + \mathrm{SIN}\left(\frac{\pi}{9}\right)\cdot \mathrm{SIN}\left(\frac{2\cdot\pi}{9}\right) + 1\right)} + \frac{\mathrm{LN}\left(2\cdot \mathrm{SIN}\left(\frac{2\cdot\pi}{9}\right) + \sqrt{3}\cdot x - 2\cdot\sqrt{3}\right)}{12\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{\pi}{9}\right)} - \frac{\mathrm{LN}\left(2\cdot \mathrm{SIN}\left(\frac{\pi}{9}\right) + \sqrt{3}\cdot x - 2\cdot\sqrt{3}\right)}{12\cdot \mathrm{SIN}\left(\frac{\pi}{18}\right)\cdot \mathrm{COS}\left(\frac{2\cdot\pi}{9}\right)}$$

2b) Example : new in *Derive 5*, all the real and complex solutions to a polynomial equation with numeric coefficients can be found to any desired degree of precision ! And the solutions returned by the equation solving commands and functions can be restricted to those solutions not involving imaginary numbers.

Let's give a good example of this, using something like the Wilkinson polynomial (from Wolfram Koepf's plenary lecture at the Second International DERIVE/TI-92 Conference in Bonn). We will try to find where the zeros of the following polynomial are located :

$$p(x) = \prod_{k=1}^{20}(x-k) - \frac{x^{19}}{10^7}$$

$$p(x) := \left(\prod_{k=1}^{20} (x - k)\right) - \frac{x^{19}}{10^7}$$

In *Derive 5*, the factory default precision has been increased from 6 to 10 digits ... but for this example (after some tries !), we need more precision :

```
APPROX(NSOLVE(p(x), x, Real), 50)

x = 4.000000000218962121204081098064548489676543328397 3 ∨ x =
  2.000000000000000000818896278314613630380378286788 76 ∨ x =
  8.928803402308404135266812000972112517793206465353 0 ∨ x =
  0.999999999999999999999999917793647533756702830441711 ∨ x =
  2.999999999999836617578358049980534631488319176439 2 ∨ x =
  20.78880561918328214760164720678366340760898281712 4 ∨ x =
  4.999999939225821559810805407621093444072922745098 0 ∨ x =
  6.999745978780573466608884327485819677153106835507 3 ∨ x =
  8.006075438449061476288682204357859442680936261422 1 ∨ x =
  6.000005824982426605421444730470333734166234688088 9
```

So, our polynomial has 10 real roots, all of them, except one, located between 0 and 10.

2c) Example : easily integrating piecewise continuous functions, *Derive* can show in a few seconds things that are much more complicated to do with other systems.

Here again, this is not a new feature of *Derive 5* (but, concerning the indicator function CHI, we can now specify the value at the transition points : however, this is not important when dealing with Fourier coefficients). We will take, here, an example from Fourier series. If f is a periodic signal of period P with Fourier coefficients

$$a_0 = \frac{2}{P}\int_P f(x)\,dx\,,\ a_n = \frac{2}{P}\int_P f(x)\cos\left(\frac{2n\pi x}{P}\right)dx \text{ and } b_n = \frac{2}{P}\int_P f(x)\sin\left(\frac{2n\pi x}{P}\right)dx\,,$$

then the energy theorem (Parseval's identity) states that

$$\frac{2}{P}\int_P f^2(x)\,dx = \frac{a_0^{\,2}}{2} + \sum_{n=1}^{\infty} a_n^{\,2} + b_n^{\,2}$$

Let us find what fraction of the energy of f is contained in the constant term and the first two harmonics. We won't take a usual signal ! Let

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 2 & \text{if } 1 < x < 2 \\ 4 - x & \text{if } 2 < x < 4 \end{cases} \qquad f(x+4) = f(x)$$

With the built-in indicator function (that now extends to include or not the limit points of the interval), it is easy to define this function :

```
f(x) := 2·x·CHI(0, x, 1) + 2·CHI(1, x, 2) + (4 - x)·CHI(2, x, 4)

f(MOD(x, 4))
```

Who wants to calculate Fourier coefficients by hand ?

$$\left[a0 := \frac{2}{4}\cdot\int_0^4 f(x)\ dx,\ a(n) := \frac{2}{4}\cdot\int_0^4 f(x)\cdot\text{COS}\left(\frac{2\cdot n\cdot\pi\cdot x}{4}\right)\ dx,\ b(n) := \frac{2}{4}\cdot\int_0^4 f(x)\cdot\text{SIN}\left(\frac{2\cdot n\cdot\pi\cdot x}{4}\right)\ dx\right]$$

The constant term and the first two harmonics carry about 99.7% of the total energy :

$$\frac{a0^2}{2} + \sum_{n=1}^{2}\left(a(n)^2 + b(n)^2\right)$$

$$\frac{84}{\pi^4} + \frac{25}{8}$$

$$\frac{2}{4}\cdot\int_0^4 f(x)^2\ dx = 4$$

$$\frac{\frac{84}{\pi^4} + \frac{25}{8}}{4}$$

$$0.9968356273$$

2d) Example : please, respect the second fundamental theorem of calculus (or the importance of being continuous : inspiration from Albert Rich/ David Jeffrey)

Some systems and integral tables give discontinuous integral to continuous integrand ... Let f be a continuous function over an interval $[a, b]$ et let F be defined by

$$F(x) = \int_a^x f(t)\ dt \quad (a \le x \le b)$$

Then, the second fundamental theorem of calculus tells us that F is an antiderivative of f: so, F is differentiable over $]a, b[$ and $\frac{d}{dx}F(x) = f(x) \quad (a < x < b)$. Trigonometric integrals calculated with the aid of the Weierstrass substitution $z = \tan\left(\frac{x}{2}\right)$ are good examples of this situation. Look at the good job done by *Derive* :

$$\int \frac{3}{5 + 4 \cdot \mathrm{SIN}(x)} \, dx$$

$$2 \cdot \mathrm{ATAN}\left(\frac{\mathrm{COS}(x)}{\mathrm{SIN}(x) + 2}\right) + x$$

$$\frac{d}{dx}\left(2 \cdot \mathrm{ATAN}\left(\frac{\mathrm{COS}(x)}{\mathrm{SIN}(x) + 2}\right) + x\right) = \frac{3}{4 \cdot \mathrm{SIN}(x) + 5}$$

It is a good job for the following reason. Let $z = \tan\left(\frac{x}{2}\right)$, then

$$\int \frac{3}{5+4\sin x} \, dx = \int \frac{3}{5+4\frac{2z}{1+z^2}} \frac{2\,dz}{1+z^2} = \int \frac{6}{5z^2+8z+5} \, dz =$$

$$2\arctan\left(\frac{5z+4}{3}\right) + C = 2\arctan\left(\frac{5\tan\left(\frac{x}{2}\right)+4}{3}\right) + C$$

Of course, if we restrict x to the interval $[-\pi, \pi]$, the last function is an antiderivative of $\frac{3}{5+4\sin x}$. But the function $\frac{3}{5+4\sin x}$ is continuous over the entire real line, so we need a continuous integral !

With a continuous integral, it is safer to apply, for example, the mean value theorem for integrals : if f is a continuous function on $[a, b]$, there is a number c between a and b such that

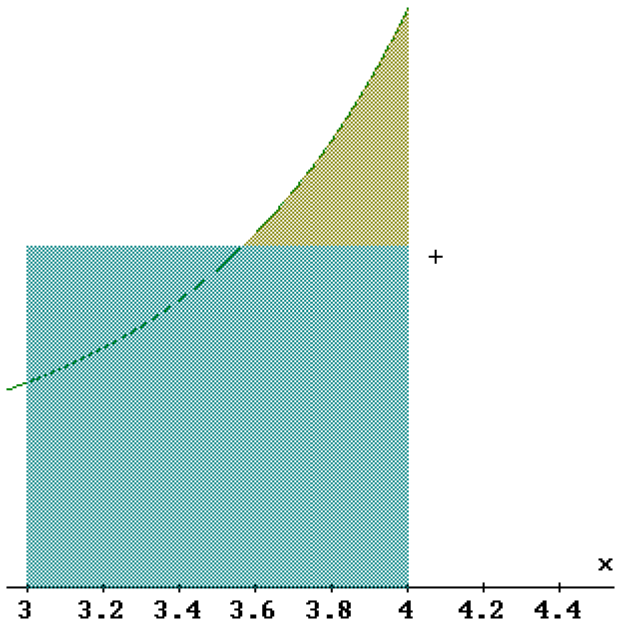
$$\int_a^b f(t)\,dt = f(c)(b-a)$$

In *Derive 5*, we can not plot Boolean combinations of inequalites and set the aspect ratio of plots. If we choose the function f at the beginning of this example and take $a = 3$, $b = 4$, we can « see » the rectangle of height $f(c)$ and length $b - a$:

$$\int_3^4 \frac{3}{5 + 4 \cdot \mathrm{SIN}(x)} \, dx$$

$$2 \cdot \mathrm{ATAN}\left(\frac{\mathrm{COS}(4)}{\mathrm{SIN}(4) + 2}\right) - 2 \cdot \mathrm{ATAN}\left(\frac{\mathrm{COS}(3)}{\mathrm{SIN}(3) + 2}\right) + 1$$

$$0.8980777888$$



2e) Example : who said that *Derive* isn't a good choice for ODEs ?

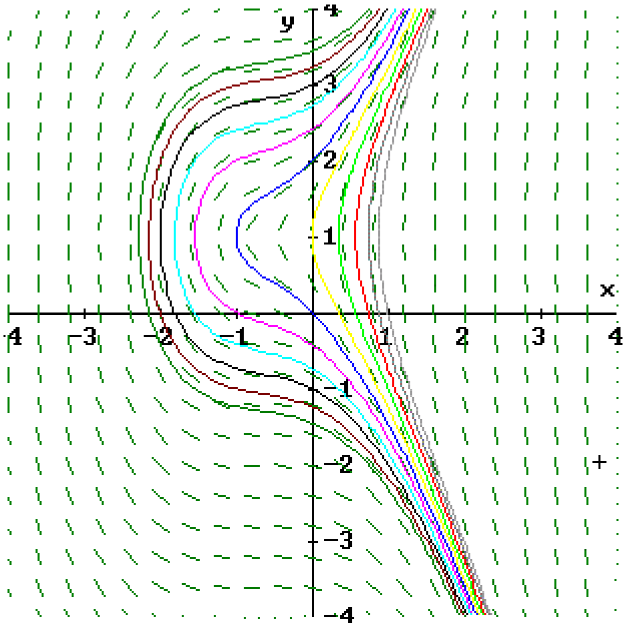
When we teach mathematics, we prove theorems like the following from Edwards/Penny concerning existence and uniqueness of solutions : suppose that the real-valued function $f(x, y)$ is continuous on some rectangle in the *xy*-plane containing the point (*a*, *b*) in its interior. Then the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(a) = b$$

has at least one solution defined on some open interval *J* containing the point *a*. If, in addition, the partial derivative $\frac{\partial f}{\partial y}$ is continuous on that rectangle, the solution is unique on some (perhaps smaller) open interval J_0 containing the point $x = a$. With a CAS like *Derive*, we can visualize this theorem : take for example this simple separable ODE

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -2 .$$

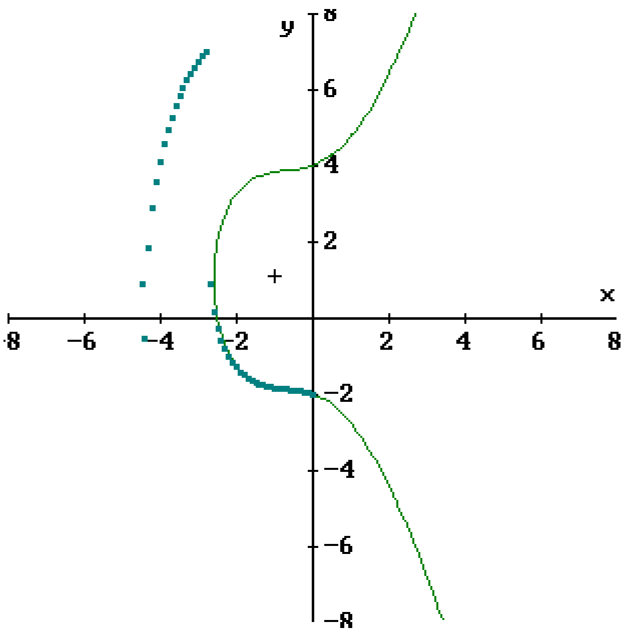
Here $f(x, y) = \frac{3x^2 + 4x + 2}{2(y-1)}$ and $\frac{\partial f}{\partial y}$ are continuous on any rectangle that does not contain the horizontal line $y = 1$. A slope field (direction_field in *Derive*) and a vector of general solutions show us rapidly the situation :



Euler's method (now EULER_ODE in *Derive* 5) gets bad when we start at (0, –2) with a negative step : the reason is simple. If we solve the ODE by hand, we get the implicit solution

$$y^2 - 2y = x^3 + 2x^2 + 2x + 8.$$

Solving for *y* gives the lower and the upper parts of the curve ; the horizontal line *y* = 1 is where the derivative approaches infinity.



If we set *y* to 1 in the above equation, we will get the value of *x* for wich a rectangle centered at (0, –2) cannot be extended to the left. This value is :

$$x = \left(\frac{\sqrt{5529}}{18} - \frac{223}{54}\right)^{1/3} - \left(\frac{\sqrt{5529}}{18} + \frac{223}{54}\right)^{1/3} - \frac{2}{3}$$

$$x = -2.578220463$$

We will quote Benny Evans for the conclusion of this example : « I have heard from time to time that *DERIVE* is not powerful enough to handle differential equations and that other software is necessary to teach at this level. I find this something of a puzzle as my own experience is that it is a perfectly wonderful tool for doing exactly that. » (The International *DERIVE* Journal, Volume 2, No. 3, page 37, 1995).

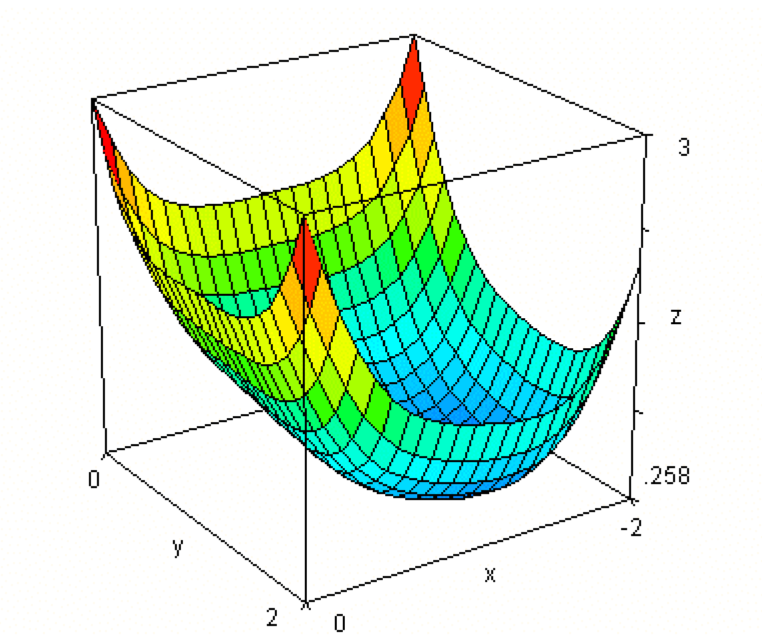
3. A better way to do multivariable calculus

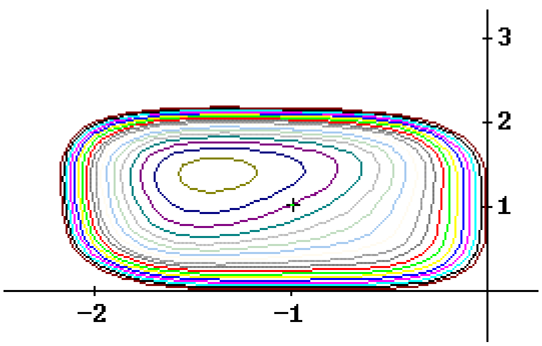
3a) Example : finding global minimum and using a CAS : don't stop thinking !

When we consider a function of two variables, we can plot the 3D graph, look at the critical points and show the contour plot. With *Derive 5*, it is very easy to explore all of these directions. A good example is given by the function

$$f(x, y) = (x+1)^4 + (y-1)^4 + \frac{1}{x^2 y^2 + 1}.$$

Let us note that the system (for the critical points) $\nabla f = (0,0)$ is not easy to solve (even if we can rewrite it in order to get two polynomial equations and try the Groebner package ...). This a good example of the importance of « thinking » when using a CAS ... Because the function f goes to infinity when we reach far from the origin, we know that the global minimum must be near the origin ! Level curves (after a 3D plot indicating the range of z) is a good tool. The 2D implicit plotting is quite fast in *Derive* since *Derive for DOS* (version 3). Because we can rotate 3D plots in real-time and determine the coordinates of points on plot surface using moveable highlighted cross lines, 3D plotting is now in the « major leagues » !





Newton's method will finish the job :

$$f(x,\ y) := (x + 1)^4 + (y - 1)^4 + \frac{1}{x^2 \cdot y^2 + 1}$$

$$\mathrm{NEWTONS}\left(\left[\frac{d}{dx}\ f(x,\ y),\ \frac{d}{dy}\ f(x,\ y)\right],\ [x,\ y],\ [-1.4,\ 1.4],\ 5\right)$$

$$\left[\begin{array}{cc} -1.4 & 1.4 \\ -1.391176296 & 1.391176296 \\ -1.391040082 & 1.391040082 \\ -1.39104005 & 1.39104005 \\ -1.39104005 & 1.39104005 \\ -1.39104005 & 1.39104005 \end{array}\right]$$

So, the coordinates of the global minimum are $x = y = -1.39$.

3b) Example : Lagrange multipliers and the substitution method and use of the new function SOLUTIONS

Many times, in multivariable calculus, we can use substitution and then return to single variable calculus. Teachers should give examples in order to make connections between subjects. Here is an easy problem that can easily be solved in many ways with *Derive* 5 : find the highest and lowest points on the ellipse generated by the intersection of the plane $x+y+z=12$ and the paraboloid $z=x^2+y^2$. It is a classical Lagrange multiplier problem with two constraints : *Derive* 5 is at work. The system defined by « eq » below is a polynomial one. It is a simple one and (maybe) *Derive 5* does not use the Groebner basis algorithm here. Anyway, the SOLUTIONS function gives the answer in the same form as the « zeros » function of the TI-92 Plus/TI-89 !

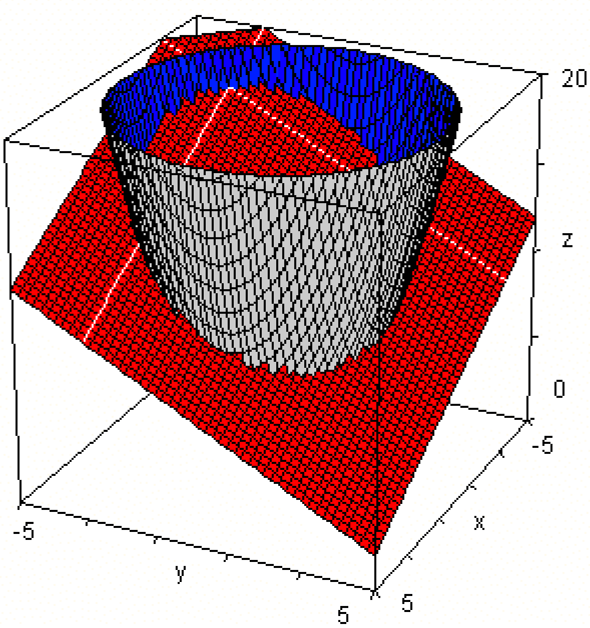
$$\left[f := z,\ g := x + y + z - 12,\ h := x^2 + y^2 - z\right]$$

$$eq := \left[\frac{d}{dx} f - \lambda\cdot\frac{d}{dx} g - \mu\cdot\frac{d}{dx} h,\ \frac{d}{dy} f - \lambda\cdot\frac{d}{dy} g - \mu\cdot\frac{d}{dy} h,\ \frac{d}{dz} f - \lambda\cdot\frac{d}{dz} g - \mu\cdot\frac{d}{dz} h,\ g,\ h\right]$$

SOLUTIONS(eq, [x, y, z, λ, μ])

$$\begin{bmatrix} 2 & 2 & 8 & \frac{4}{5} & -\frac{1}{5} \\ -3 & -3 & 18 & \frac{6}{5} & \frac{1}{5} \end{bmatrix}$$

We want to see these points ! Let's see the maximum value (in version 5, we can display multiple plots with unique color tiling) :



Fourth International *DERIVE*-TI89/92 Conference

We said that the intersection curve of the two surfaces is an ellipse. Let's use parametric 3D plot : first, we need to obtain parametric equation. The projection in the *xy*-plane of this ellipse is the circle $x^2+y^2=12-x-y$ or $\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=12.5$. So we can set

$$\begin{cases} x=-\frac{1}{2}+\sqrt{12.5}\cos t \\ y=-\frac{1}{2}+\sqrt{12.5}\sin t \end{cases}$$

and, because $z=x^2+y^2=12-x-y$, the third component of our parametric curve is simply $12+\frac{1}{2}-\sqrt{12.5}\cos t+\frac{1}{2}-\sqrt{12.5}\sin t$.

$$x := -\frac{1}{2} + \sqrt{12.5}\cdot\mathrm{COS}(t)$$

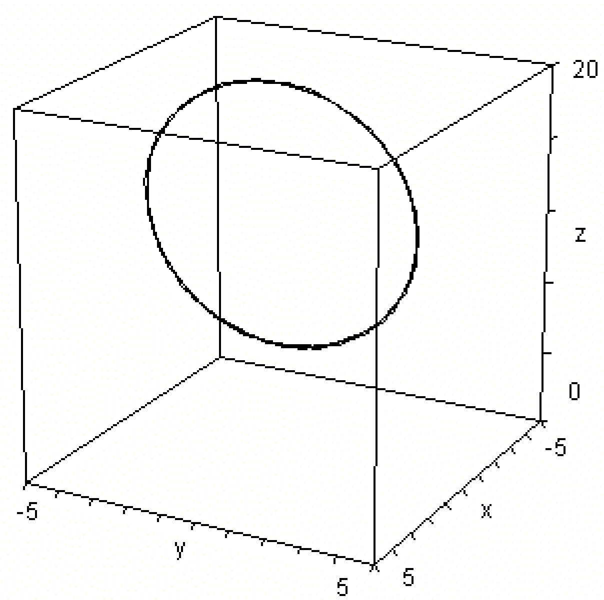
$$y := -\frac{1}{2} + \sqrt{12.5}\cdot\mathrm{SIN}(t)$$

$$z := 12 - x - y$$

$$-\frac{5\cdot\sqrt{2}\cdot\mathrm{COS}(t)}{2} - \frac{5\cdot\sqrt{2}\cdot\mathrm{SIN}(t)}{2} + 13$$

[x, y, z]

Again, in version 5, parametric expressions (curves, surfaces) can be plotted easily :

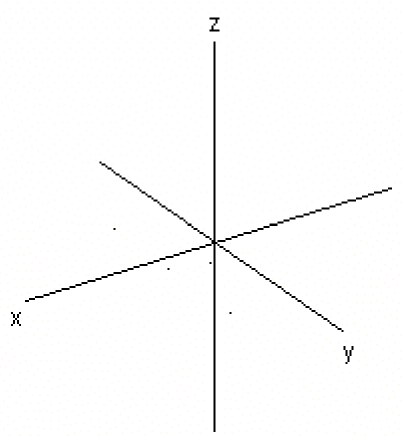


Of course, the maximum and minimum values of the function $13-\sqrt{12.5}\cos t-\sqrt{12.5}\sin t$ over the interval [0, 2π] is another way to confirm the answer.

3c) Example : parametric surfaces and the teacher's place in teaching !

Derive 5 does not support 3D implicit plots. Well, we can use parametric 3D plot to overcome this lack. Here, we will take an example that uses many things. Sometime ago, I gave my students the following problem : find the equation of a sphere that goes through the four following points :
(2, 5, –3), (8, –1, 3), (6, 3, 1) and (3, 4, 1).

This question came after a study of Gauss-Jordan's algorithm. Not many students noticed that we can solve this problem using a matrix (solving a linear system, not a polynomial second degree system). With *Derive* 5, we can use the two approaches and even more, plot the points and, using spherical coordinates, plot the sphere !



If we start with the equation $(x-a)^2+(y-b)^2+(z-c)^2=r^2$, then an expansion gives $x^2+Ax+y^2+By+z^2+Cz=D$ where $A=-2a$, $B=-2b$, $C=-2c$ and $D=r^2-a^2-b^2-c^2$. We show that the center of the sphere is at the point (2, –4, –3) and the radius is 9, so the equation of the sphere is simply $(x-2)^2+(y+4)^2+(z+3)^2=81$. And, because of the spherical coordinates, we can parametrize this sphere with

$$\begin{cases} x=2+9\cos\theta\sin\phi \\ y=-4+9\sin\theta\sin\phi \qquad -\pi\le\theta\le\pi,\ 0\le\phi\le\pi \\ z=-3+9\cos\phi \end{cases}$$

Here again, a superb job done by *Derive* 5 :

$$\begin{bmatrix} [2, 5, -3] \\ [8, -1, 3] \\ [6, 3, 1] \\ [3, 4, 1] \end{bmatrix}$$

[A :=, B :=, C :=, D :=]

[a :=, b :=, c :=]

$f1(x, y, z) := x^2 + A \cdot x + y^2 + B \cdot y + z^2 + C \cdot z = D$

$f2(x, y, z) := (x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$

SOLVE([f1(2, 5, -3), f1(8, -1, 3), f1(6, 3, 1), f1(3, 4, 1)], [A, B, C, D])

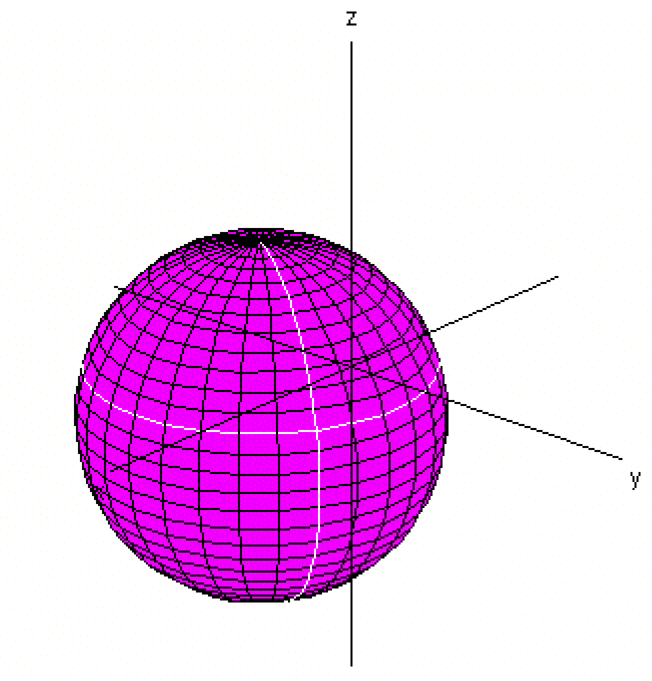
$[A = -4 \wedge B = 8 \wedge C = 6 \wedge D = 52]$

SOLVE([f2(2, 5, -3), f2(8, -1, 3), f2(6, 3, 1), f2(3, 4, 1)], [a, b, c, r])

$[a = 2 \wedge b = -4 \wedge c = -3 \wedge r = 9, a = 2 \wedge b = -4 \wedge c = -3 \wedge r = -9]$

And, finally, we plot the sphere :

$[2 + 9 \cdot COS(s) \cdot SIN(t), -4 + 9 \cdot SIN(s) \cdot SIN(t), -3 + 9 \cdot COS(t)]$



3d) Example : shading a region for a double integral

Let a thin plate occupy the region R in the first quadrant common to the circle of radius 1 centered at (2, 0) and the circle of radius 2 centered at (0, 0). Suppose that the density function is $\delta(x,y) = x^2 y^4$ (in grams per square centimeters). Where is the center of mass located ? We have to use the following formulas for the mass M and center of mass $(\bar{x}, \bar{y})$:

$$M = \iint_R \delta(x,y)\, dA \; ; \; \bar{x} = \frac{1}{M}\iint_R x\delta(x,y)\, dA \; ; \; \bar{y} = \frac{1}{M}\iint_R y\delta(x,y)\, dA$$

Of course, plotting the region and setting a double integral is useful !

$$\text{SOLVE}((x-2)^2 + y^2 = 1 \wedge x^2 + y^2 = 4, [x, y])$$

$$\left(x = \frac{7}{4} \wedge y = -\frac{\sqrt{15}}{4}\right) \vee \left(x = \frac{7}{4} \wedge y = \frac{\sqrt{15}}{4}\right)$$

$$\text{SOLVE}((x-2)^2 + y^2 = 1, x)$$

$$x = 2 - \sqrt{(1 - y^2)} \vee x = \sqrt{(1 - y^2)} + 2$$

$$\text{SOLVE}(x^2 + y^2 = 4, x)$$

$$x = -\sqrt{(4 - y^2)} \vee x = \sqrt{(4 - y^2)}$$

$$(x-2)^2 + y^2 \le 1 \wedge x^2 + y^2 \le 4 \wedge y \ge 0$$

$$\text{mass} := \int_0^{\sqrt{15}/4} \int_{2-\sqrt{(1-y^2)}}^{\sqrt{(4-y^2)}} x^2 \cdot y^4 \, dx\, dy$$

$$x\_ := \frac{1}{\text{mass}} \cdot \int_0^{\sqrt{15}/4} \int_{2-\sqrt{(1-y^2)}}^{\sqrt{(4-y^2)}} x \cdot x^2 \cdot y^4 \, dx\, dy$$

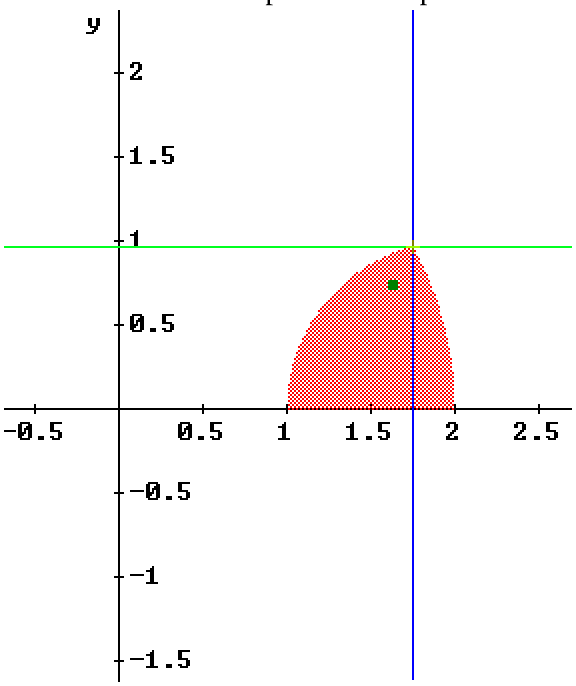
$$y\_ := \frac{1}{\text{mass}} \cdot \int_0^{\sqrt{15}/4} \int_{2-\sqrt{(1-y^2)}}^{\sqrt{(4-y^2)}} y \cdot x^2 \cdot y^4 \, dx\, dy$$

[x_, y_]

$$\left[\frac{5\cdot\left(42869\cdot\sqrt{15}-401408\cdot\mathrm{ATAN}\left(\frac{\sqrt{15}}{5}\right)\right)}{16\cdot\left(858368\cdot\mathrm{ATAN}\left(\frac{\sqrt{15}}{5}\right)-229376\cdot\pi+35131\cdot\sqrt{15}\right)}, -\frac{29838521}{2160\cdot\left(858368\cdot\mathrm{ATAN}\left(\frac{\sqrt{15}}{5}\right)-229376\cdot\pi+35131\cdot\sqrt{15}\right)}\right]$$

[1.635040946, 0.7336293033]

We show below the point that represents the center of mass :



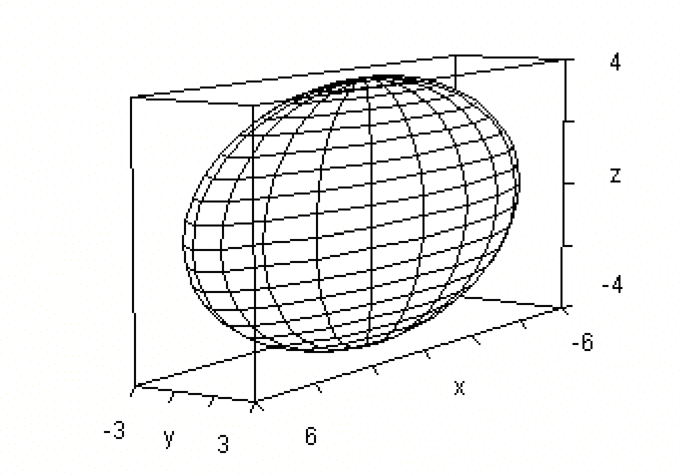
3e) Example : finding the maximum volume of a closed rectangular box with faces parallel to the coordinates planes inscribed in the ellipsoid

$$\frac{x^2}{36}+\frac{y^2}{9}+\frac{z^2}{16}=1$$

This final example will use some of *Derive 5* new features. If we pick the first octant corner of the box we are looking for and call it $[x, y, z]$, then our problem is to maximize the function $f = 8xyz$ subject to the constraint $g = 0$, where $g = \dfrac{x^2}{36} + \dfrac{y^2}{9} + \dfrac{z^2}{16} - 1$. Let see the constraint in 3D, with the aspect ratio : 9, 2, 4. In order to plot, we used parametric plot

```
[6·COS(s)·SIN(t), 3·SIN(s)·SIN(t), 4·COS(t)]
```

with $-\pi \leq s \leq \pi$ and $0 \leq t \leq \pi$:



We use Lagrange multiplier again :

$$f := 8 \cdot x \cdot y \cdot z$$

$$g := \frac{x^2}{36} + \frac{y^2}{9} + \frac{z^2}{16} - 1$$

$$lag := \left[\frac{d}{dx} f - \lambda \cdot \frac{d}{dx} g,\ \frac{d}{dy} f - \lambda \cdot \frac{d}{dy} g,\ \frac{d}{dz} f - \lambda \cdot \frac{d}{dz} g,\ g\right]$$

```
ma := SOLUTIONS(lag, [x, y, z, λ])
```

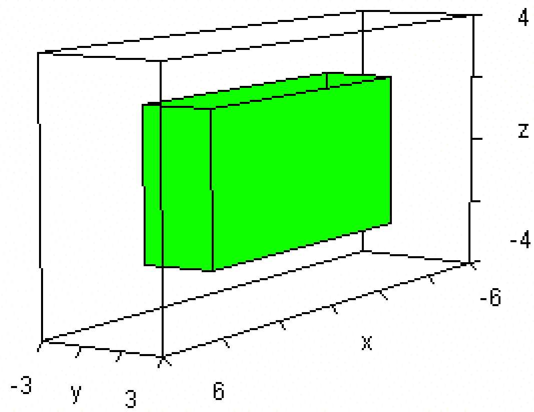
Here, the Groebner package is very useful (even if we can solve by hand ...) ! There are 14 solutions !

```
DIMENSION(ma) = 14
```

The one we are looking for is the only positive one (for x, y and z ; and λ is positive too) :

$$\left[2 \cdot \sqrt{3},\ \sqrt{3},\ \frac{4 \cdot \sqrt{3}}{3},\ 96 \cdot \sqrt{3}\right]$$

In *Derive 5*, we can plot matrices of data points. Here is the box we are looking for :



4. Why I like *Derive* so much

I'm thinking of an example from Larry Gilligan (First *Derive* conference in Plymouth). Earlier versions of *Derive* were not able to solve a quadratic equality, for example, as simple as

$$|x^2+3x-1| \le |2x+1|.$$

As we can see, *Derive* 5 solves easily this inequation :

$$\text{SOLVE}(|x^2 + 3\cdot x - 1| \le |2\cdot x + 1|, x)$$

$$-5 \le x \le -2 \vee 0 \le x \le 1$$

Larry said that it was not important if earlier versions of *Derive* were unable to solve this inequation, we just have to plot, in the same window, the graphs of $|x^2+3x-1|$ and of $|2x+1|$ and we will find the solutions ! ! !

With new versions, Computer Algebra Systems add new features. But what remains the most important is that they remain « simple », easy to use. The user must always be confident with such a system. *Derive* 5 : The Easiest ... Just Got Better !