Fourth International Derive TI-89/92 Conference
Liverpool John Moores University, July 12 – 15, 2000

Main Notions and Achievements of Modern Nonlinear Dynamics

Elena D. Surovyatkina, Sergey V. Biryukov*
Kamchatka State University, Russia
Email: elena@marine.kamchatka.su
***Moscow Pedagogical State University, Russia**
Email: svb3@hotmail.ru

Abstract

This workshop is an improved version of our workshops at the 2rd and 3rd International DERIVE and TI-92 Conferences [1,2]. It includes the introductory, well tested part, and presents new scientific results in the last part. So, we hope it to be interesting and useful for newcomers and also for those who attended our previous workshops. An actual chaos predictability discussion is included.

Introduction

The attractor notion can be illustrated at the example of the two maps shown below. They are represented by the simplest of non-linear equations:

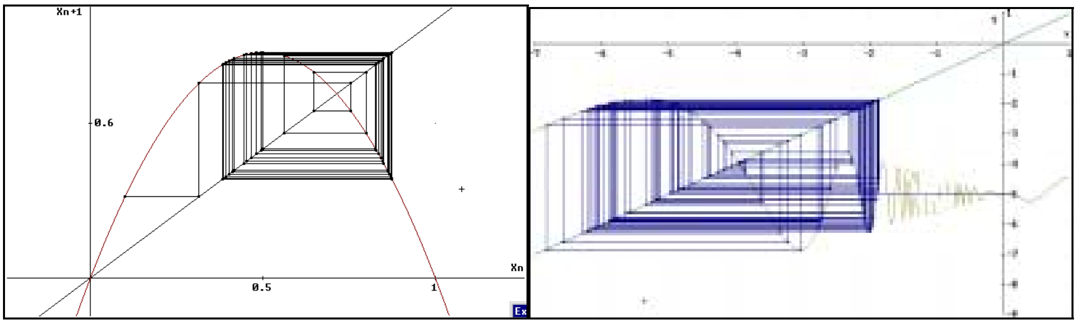
$$x_{n+1} = r\, x_n\, (1-x_n), \qquad (1)$$

$$x_{n+1} = r + x_n \cdot e^{-\frac{d}{N}} \cdot \cos\left(\frac{2\pi}{N(1+\beta \cdot x_n)}\right), \qquad (2)$$

where $n=1,\ldots,N$, and x_n is the value under consideration, the so-called iterative value. These equations are nonlinear and their solutions have very interesting behaviour for some values of the control parameter r. We shall see that for some particular values of r the equations' results are chaotic solutions. Theoretical foundations of the problems are analysed in detail in papers [1-3], which contain voluminous bibliographies.

If the result of the map does not change with iterations, then we say that the system is in *a stationary state*. A fixed point is a value of x such as $x_n = x_{n+1} = x^*$ for some n. In this case the limit of x is called *an attractor*.. The periodic solution with period 2 (2 consequently changing constant values x_n and x_{n-1} is called an *attractor of period two*. This name *attractor* is obvious, because any sequence started from an arbitrary point in the domain of definition will reach this (these) value(s) by definition.

We used logistic map (1) as an example in our report at the 3d International DERIVE and TI-92 Conference [5] (Fig. 1a.) It will be also used in the second part of our paper for chaotic systems predictability study. The so-called multi-modal map (2) is a more complex one. If the control parameter r is in the interval [-10,6] (except r=–9.1, and r=0.58, where there is a break in the map), 2 cascades of period doubling bifurcations from period 1 to chaos can be observed (Fig.2b).



a b

Fig.1.Attractors: a) Logistic (r=3.5, x0=0.1); b) Multimodal (r=-5)

1. Horizon of predictability for chaotic processes

The ability to forecast a process progressing in time, as researchers in natural sciences say, time series, is very much called for in different areas of human nature.We shall describe some approaches to the problem of dynamical forecasting. The given forecasting is based on certain dynamical laws.

The idea about strange attractors as the gigantic amplifier is well shown in several research works. In local unstable state systems the little fluctuations are increasing in accordance with exponential law and they are reaching a limiting value that can be compared with the dimension (amplitude)of the attractor, *A*.

Taking *A* to be the swing (typical magnitude) of oscillations under study and δ the perturbation in the system occurring due to one or the other factor, the time of predictable behaviour τ_{pred} can be written as (see, for instance, Ref. [1]):

$$\tau_{pred} = \frac{1}{2\lambda^+} \ln \frac{A}{\delta}, \tag{1}$$

where λ^+ is maximum Lyapunov exponent giving the perturbation growth rate.

Its maximal value τ_{pred} corresponds to minimal achievable value of inaccuracy δ (low instrumental noise, low ambient noise, good model for predictability). It was named the horizon of predictability [2].

2. DELAYED CORRELATION BETWEEN NOISE AND FORECASTING ERROR

The prevailing reason for prognosis error grows (inherently) with the noise in the system that is under study. The question to be discussed here is the establishment of an interrelationship between prediction of the errors at a given moment and inherent noise, driving the system BEFORE that time.

We studied the noisy logistic map (1) and numerically derived a correlation function $R_{uf}(\tau)$ between the noise $f(t-\tau)$, acting at time τ before t, and the forecasting error, $u(t) = y(t) - z(t)$, where $y(t)$ is observed and $z(t)$ is the predicted process.

Thus,

$$R_{uf}(\tau) = \langle u(t) f(t-\tau) \rangle \qquad (3)$$

The correlation function (3) for the logistic map (r=3.5, x0=0.1, n=60), shown in Fig.2 has a tooth-like shape. Splitting the data into 4 groups in accordance with the attractor “corners” result for more smooth curves (Fig.3). The same plot in a linear - logarithmic scale (Fig.4) shows 2 exponents: the left growing Lyapunov exponent, and the right decreasing one.

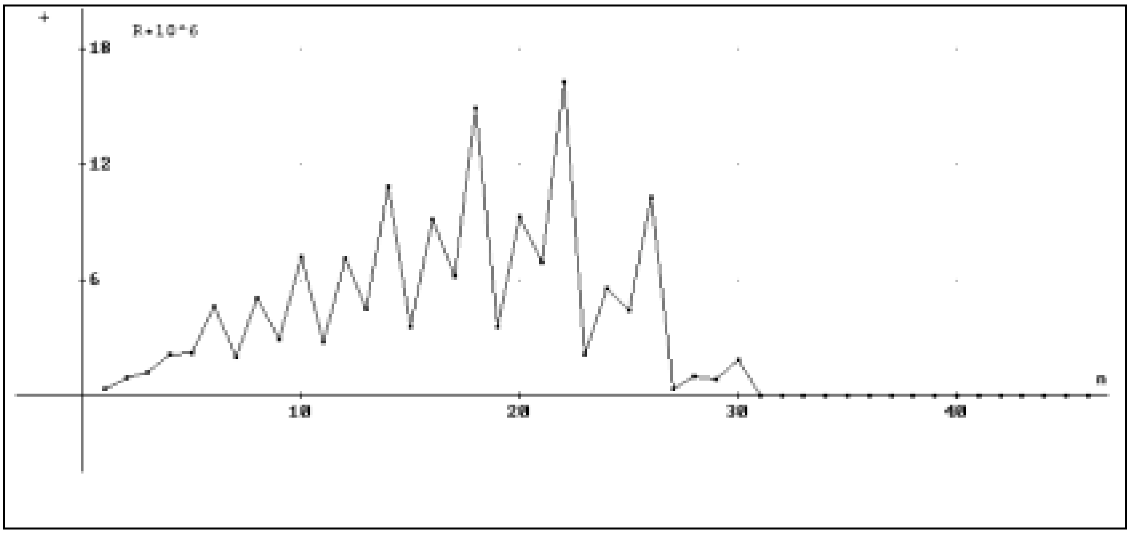


Fig. 2. Delayed correlation <noise-prediction error> in logistic map: control parameter r=3.5, noise level 10^{-4}

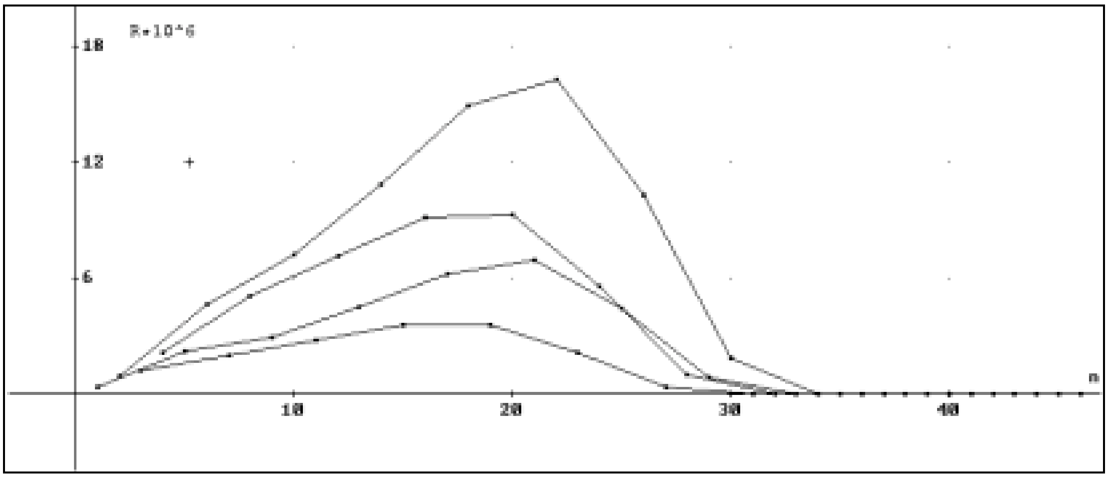


Fig. 3. The above split into 4 and thus more smooth. The maximum of the correlation function corresponds to the predictability time.

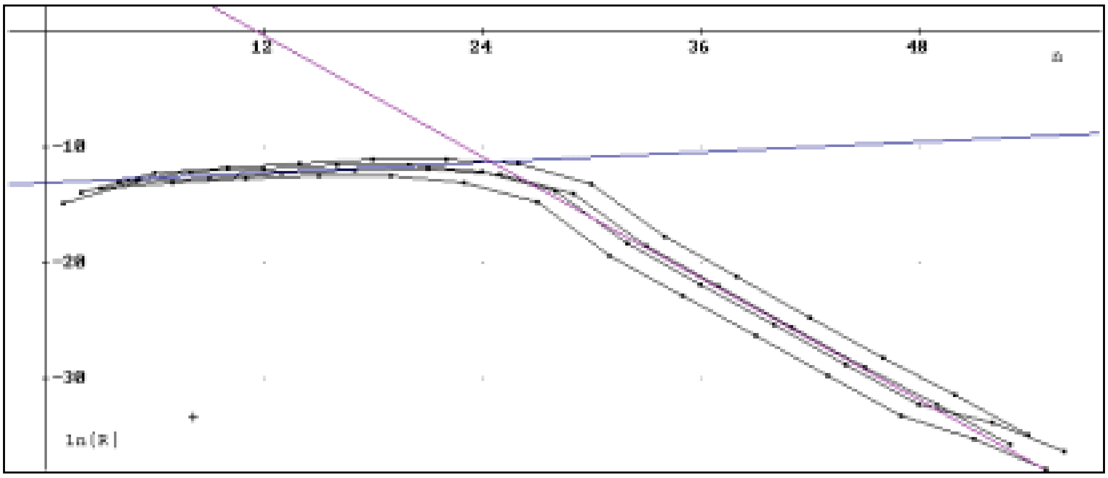


Fig. 4. The above in lin-log scale. The left line slope is defined by the Lyapunov exponent.

The workshop is supported with:

- theoretical background (this paper),
- paper worksheet with the description of main and additional exercises,
- electronic DfW-worksheet,
- MTH utility file for main calculations.

Exercises include:

- size of the logistic and multimodal maps determination;
- error correlation function R for the logistic map
  - plotting,
  - splitting,
  - scaling, and
  - fitting;
- predictability time and Lyapunov exponent determination.

Additional homework exercises include:

- the maps' starting point role,
- 1st and last points of the lin-log plot peculiarities (far from the exponent),
- exponents powers ratio dependence on the logistic map parameter r,
- multimodal map analysis.

3. Conclusions

The above examples show, that DERIVE can be successively applied to the numerical modelling of complicated nonlinear systems. DERIVE is a very prospective tool for student scientific research. The high precision of calculations supported by DERIVE provide a unique opportunity for the investigation of chaos in nonlinear systems.

4. References

1. *Shaw, R.S.*. Zeitschr. Fur Naturfoschung, 36a (1),80 (1981)
2. *Lighhill, J.*. Proc. Royal Soc., London A407,35 (1981)
3. *Anosov, L.A., Butkovskii, O.Ya., Kravtsov, Yu. A. Surovyatkina, E. D.*. Predictable Nonlinear Dynamics: Advances and Limitations. In: Chaotic, Fractal, and Nonlinear Signal processing, Mystic, CT July 10-14 ,1995,American Institute of Physics.-C. 71-91 (1996)
4. *Butkovsky O. Ya,. Biryukov S. V., Surovyatkina E. D.* Derive Application to Nonlinear Dynamic Systems. 2rd International Derive and TI-92 Conference, Computer algebra in Math education, Bonn, July 2-6, 1996, p. 483-487.
5. *Surovyatkina, E.D.,* Chaos investigation with DERIVE. 3rd International Derive and TI-92 Conference, Computer algebra in Math education, July 14-18, 1998.