Fourth International Derive/TI-92 Conference

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New Technologies – New Means of Mathematics Teaching

A Comenius Project

Josef Böhm
Pedagogical Institute of Lower Austria, Austria
Email: nojo.boehm@pgv.at

In spring 1997 I attended an information day at the Austrian Sokrates Agency about possibilities for launching an EU-project in the frame of one of the running programmes SOKRATES, LINGUA or LEONARDO. After a discussion, I was asked by one of the staff members if I would like to initiate an EU-project in the frame of the Comenius 3.1 programme. As I have many connections reaching out of Austria to mathematics teachers who are busy in school, with in- and pre-service training for teachers and who are as fascinated as I am by the possibilities and opportunities of changing mathematics teaching using the many new technologies like PC, Computer Algebra Systems, Dynamic Geometry, Computer Based Laboratories and very advanced pocket calculators, I didn´t need a long time to agree. So I asked Dr. Franz Surböck, the head of department BHS (Berufsbildende Höhere Schulen = Vocational Secondary Schools) in the Pedagogical Institute of Lower Austria (PI-NÖ), if his institution would be interested in coordinating a project called "Modern Technologies in Teaching Mathematics - MTTM". The PI-NÖ is responsible for the pre- and in-service training for teachers in Lower Austria. This institution had successfully coordinated another EU-project some time ago. So Dr. Surböck agreed but he also pointed out that I would load a burden of work on my shoulders. Thanks to modern technologies like email, I very quickly found excellent partners: Bärbel Barzel from Germany, Tony Watkins from England, David Sjöstrand from Sweden and Paul Drijvers from the Netherlands, all of them very experienced teachers and teacher trainers and well known to me and each other for many years.

Supported by the Austrian SOKRATES Buro, we applied for a Preparatory Visit to plan the project. Very late – one day before Christmas holidays – we received the approval and in the first days of January 1998 we all met in Lower Austria. In the meanwhile, we were joined by Dirk Janssens from Belgium by email only and thus we had a new partner. It was a challenge for us all – members of six European countries – to face the documents and to fill in all columns of the application form for the project. These were two very hard days and there were several occasions when one or the other of us could be heard muttering, 'Let it be, let me go home'. But then someone else would say, 'Let´s try! We want to do it and we will do it'. And finally we did it.

We applied for the project Modern Technologies in Teaching Mathematics (MTTM), and summarized our goals as follows:

> Production of a multilingual information pack of materials suitable for in-service training of mathematics teachers at Secondary Schools (grades 8 to 13) under special consideration of the use of modern technologies in teaching mathematics in EU countries.
>
> The target group is school mathematics advisors, heads of school mathematics departments, senior teachers, teachers for pre- and in-service training, and others who can be agents of change within their local environments. Students will benefit from

better trained teachers because they will improve their learning results and acquire additional competencies.

I delivered the application just in time and then we waited until the autumn of 1998 to receive approval for our project. We collected materials and exchanged papers by email and surface mail and prepared the first meeting in January 1999 in St. Pölten, Lower Austria.

This meeting was a very fuitful one. We discussed the collected materials and fixed a structure for the whole collection and of each contribution as well. Some important points were - among others:

- our target group are teachers: motivated but not experts;
- the examples must be "easy" enough to attract the average teacher;
- we will have small units and one extended teaching sequence;
- the materials should be independent of special soft- and hardware products (hints can be provided separately);
- a good teacher training must be a good example of teaching;
- the units should form a sort of library and/or store;
- we will include classroom experience, a concept of a training course and additional tasks and questions for the teachers to provoke them to reflect on their own and their students´ activities.

We fixed a time-table and the duties of all the group members in order to have a successful meeting in Düsseldorf during June 1999, organized by Bärbel Barzel. The following months were busy for all of us. Beside our daily work as teachers, lecturers, researchers, mother, father etc, we wrote papers and comments, sent emails and documents across Europe; revised and improved our papers and tried to find additional interesting materials in our home countries.

When we met in June we had almost all the papers together. We discussed each of them and made some changes. We missed only one promised contribution. Eberhard Lehmann wanted to bring his Module - Principle into a form agreeing with that of our units and this required some additional time. But as you can see in the book, he was on time and we are happy to have his "Brick onto Brick" among our products.

The deadline was the end of June to bring the papers into camera ready form. We had to find a publisher and to organize some public relations for our product. We will try to produce a German translation and we hope to have a French and a Dutch version, too.

Finally we agreed on a new title which, in our opinion, expresses the aim of our project much better than MTTM:

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New Technologies – New Means of Mathematics Teaching

My work as "coordinator" took place on two levels: the specific technical and mathematical, with which I am familiar, and the administrative, which was a very new field of experience for me. But thanks to the help of more experienced group members and Mr Surböck I was able to master this task.

Working together in the frame of this project brought a lot of benefit to us all. With respect to our work as teachers and teacher trainers we achieved insight into the problems of other countries, and with respect to our social contacts we were all able to bring in our full personality and simultaneously to submit ourselves to our common goal. Paul Drijvers expressed our feelings in a talk at the dinner table very precisely and I asked him to summarize his opinion, which is our opinion:

> 'Cooperating in the MTTM team has been a fascinating process. As our aim was to produce a booklet with concrete student examples in a format that enables its use for teacher training purposes, we needed to discuss our pedagogical and educational points of view in detail. Understanding each other at this level is not obvious. Besides the language barriers that the six participants from different countries had to overcome, there are other factors that are at least as important: differences in educational system, mathematics curriculum, classroom culture, pedagogical philosophy and opinions on the use of technology in secondary education. The product being a coherent resource for teacher training indicates that the team was able to overcome the obstacles; to the team members this process was an important experience.'

We all hope that this booklet can transfer inspiring and motivating ideas about how to change teaching using modern technologies in order to have more interesting, more lively and more open mathematics education for everyone who likes to work with young people in general and who particularly like to teach mathematics.

At last I have to thank the staff of the Austrian SOKRATES Buro for their support together with Dr Surböck from the PI-Nö for his help and patience. I am very grateful to my five partners and their institution heads for their amicable cooperation and I hope that we will have other opportunities and challenges to face together in the future.

We would especially like to thank all the colleagues who contributed to this package and helped to give it a real European dimension: Reimund Albers (Unit 7.1), Hubert Weller (Unit 6.1), Eberhard Lehmann (Unit 1.3), Guido Herweyers and Luc Gheysens (Unit 4.1) together with many others mentioned in the references to the units.

At the preparatory visit we very much appreciated the advice and support of Prof. Manfred Kronfeller (Technical Univeristy of Vienna), Mr Helmut Heugl (Landesschulrat Noe) and Mr Peter Schüller (Austrian Ministry of Education). Mr Heugl and Prof. Kronfellner, both being experts in mathematics education and inservice training, were willing to evaluate our materials internally.

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We are very grateful that colleagues from Austrian Secondary Schools agreed to test some units in a seminar specially organized for that purpose. Their comments were very helpful for us to improve our texts.

At last we have to thank our family members for their support and patience during one year. We were absent from home for some time physically and much more time we were absent mentally, sitting in front of our computers or reading texts. Without their understanding and help we would not have been able to finish this work in time.

The partners (in alphabetical order):

Bärbel Barzel – ZKL, University of Muenster, Germany

Josef Böhm – PI of Lower Austria, Austria (Coordinator)

Paul Drijvers – Freudenthal Institute, University of Utrecht, Netherlands

Dirk Janssens – Catholic University of Leuven, Belgium

David Sjöstrand – Chalmers University Gothenburg, Sweden

Anthony Watkins – University of Plymouth, England.

The project´s time table summarized:

Application for a Preparatory Visit on 26-02-1997

Acknowledgment of Receipt on 0404-1997 (Information about acceptance was announced for June 1997)

Acceptance came in late fall (with the order to hold the visit beween July and October – then until 31 January).

Deadline for project application was 31-01-1998. The project should run from 01-09-98 to 31-08-99.

Preparatory visit: 23-01-1998 – 25-01-1998.

Application delivered on 30-01-1998

Notification of acceptance on 23-10-1998

1. Meeting: 7 – 10 January 1999 in St.Pölten (Lower Austria)
2. Meeting: 3 – 6 June 1999 in Stenden (Germany)

Printed in August 1999

End of project 31-08-1999

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At the Prep Visit we fixed our goals, the target group and the expected outcomes.

The first meeting fixed the main ideas, the contents and - very important – the structure and format of the units in the package:

The table of contents and the common structure of the units are given below:

Contents

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Title of the Unit

Pedagogical Issues

Representation(s)	Investigation	WhiteBox <=> BlackBox	Documentation
Numerical Table Graphical	Guided Open		Notes Worksheet Written Report, ...

Platform Issues

Spreadsheet	Graphics Calculator	CAS	Dynamical Geometry

Aim and Description of the Unit

Role of the Technology

- Investigating Tool
- Visualization Tool
- Calculating Tool
- Example Generating Tool
- Practice Tool (Trainer)
- Checking Tool (Controller)

Preliminary Knowledge Required

Duration of the Unit

Classroom Organization

Statement of the Student Example

Hints for the use of the TI-89 or TI-92 (or DERIVE or Excel)

What do you want? *How can you do it?*

Proposed Solution(s)

Example(s) of Students´ Working

Questions and Tasks for the Teacher

(*eg* - *How would you assess the student´s work here?*
- *What documentary conclusions are appropriate here?*
- *.........)*

Supplementary Examples for Assessment

References

See now one sample unit:

4.1 Geometrical Optimization Problems

Pedagogical Issues

Representation(s)	Investigation	WB <=> BB	Documentation
Numerical Graphical Analytical Table Geometrically	Open Guided	WhiteBox → BlackBox	Notes Worksheet Written Report Poster Oral Presentation

Platform Issues

Spreadsheet	Graphics Calculator	CAS	Dynamical Geometry
	X	X	X

Aim and Description of the Unit

This unit deals with geometrical optimization problems.

We consider four different representations:

- the classical analytical approach, using derivatives,
- the graphical approach,
- the numerical method using tables,
- the dynamic geometrical illustration.

Students should learn to handle these different methods to solve the problem.
The introduction of a parameter makes it possible to generalise the solution but also to get deeper insight in the problem.
Technology can help to focus on the influence of the parameter to the solution.

By choosing another independent variable to describe the problem, students can discover that this can simplify significantly the analytical representation. Once the solution is found, it is interesting to look at the geometry of the situation and explain this solution by purely, but convincing, geometrical arguments.

With this unit students will explicitly reflect on some important problem solving techniques.
Discussion about assessment of these different aspects and of the way technology is used by the students is the objective of the second part of the unit.

Role of the Technology

The technology device is used in the following modes:

- Investigating Tool
- Visualization Tool
- Calculating Tool
- Table Generator
- Checking Tool

Preliminary Knowledge Required

Contents

Students need to know elementary geometry and trigonometry.
The equation of a line is needed for the second problem.
Students should know how derivatives work as a tool to find an extremum.

Duration of the unit

Minimum two lessons in the classroom.

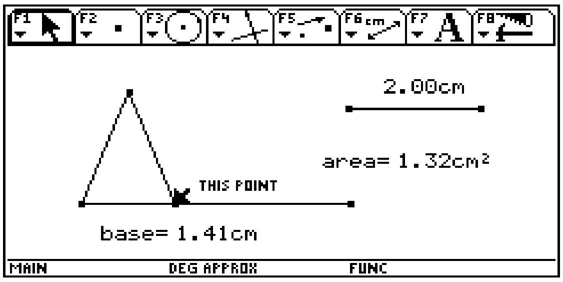
Classroom Organization

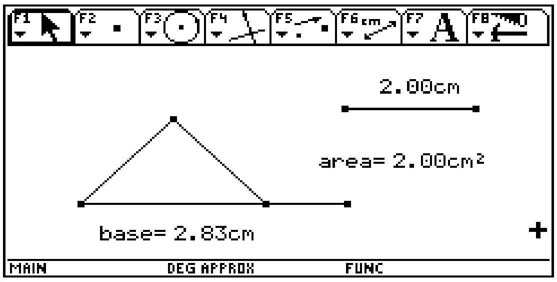
The teacher demonstrates the first problem using dynamic geometry.
The students can work in pairs for the analytical, graphical and numerical ways to find the solution. The second problem is an assesment task for students and can be organised in different ways. It can also be presented as a “research-task”.

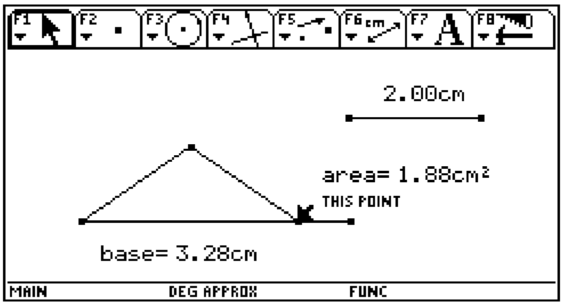
Statement of the Student Example

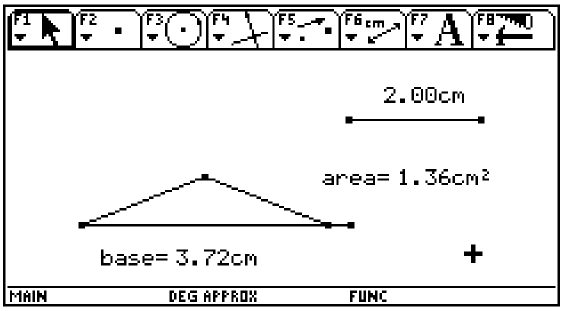
Consider all isosceles triangles with legs equal to 2, which one has got the largest area?

1. We first illustrate the problem with dynamic geometry. Read off the largest area. What is the shape of the largest triangle?

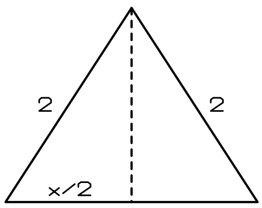








2. Express the area A of an isosceles triangle, with legs equal to 2, in terms of its base x.



3. What is the domain of the area function? Draw the graph of the function with the appropriate viewing window. Find out the maximum value, using trace or another available tool.

4. Make a table of areas for different values of x in the domain of the function, find the largest area.

5. Check your result analytically and find the maximum using derivatives. A computer algebra system may be used to compute the derivative.

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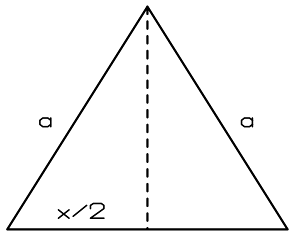
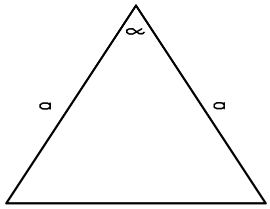
6. You probably found that the maximum area is 2, the length of the legs being 2 also. Is 3 the maximum area if the equal legs of the triangele have length 3? Generalize the problem for a triangle with legs of length a.



7. The shape of the triangle with maximum area was the same for legs 2 and legs 3: the top angle is special. Verify this observation also in the general case. We can use the top angle of the triangle as independent variable. In order to investigate this, first find the area of an isosceles triangle with legs equal to a, making an angle α at the top in function of a and α.



8. For which value of α do we have a maximum area? What is the maximum area?

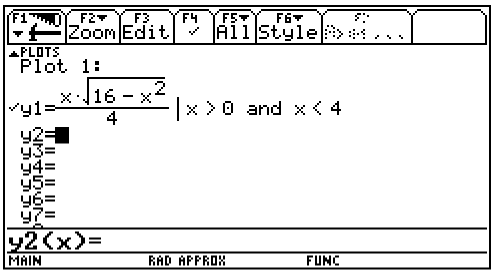
9. We expressed the area of the isosceles triangle in terms of its base or its top angle. Working with the top angle results in a simple function! What are the other possible choices of independent variables to describe the problem?

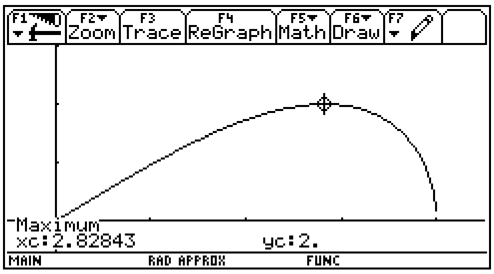
Proposed solutions

1. The dynamical geometry suggests that the maximum area is 2, at the moment when the triangle is right angled. This can be confirmed by measuring the top angle dynamically.

2. $A = \dfrac{x.\sqrt{16-x^2}}{4}$.

3. The domain of the function is the open interval (0,4). We take the viewing window $[-0.5, 4.5] \times [-0.5, 3]$.





4. Make successive tables, reducing the increment of the independent variable:

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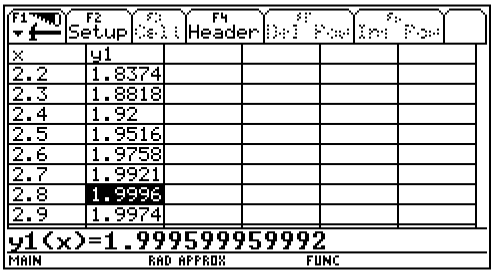

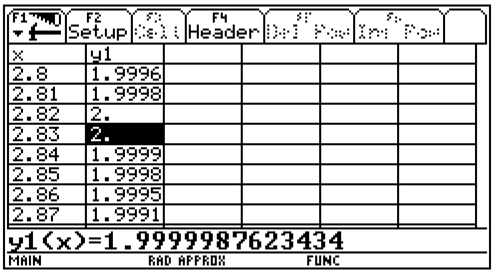

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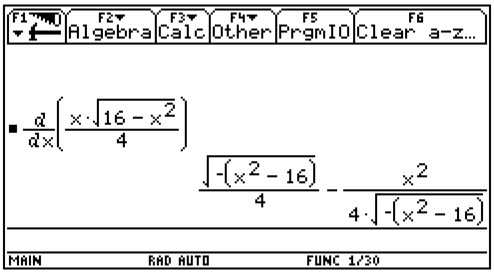
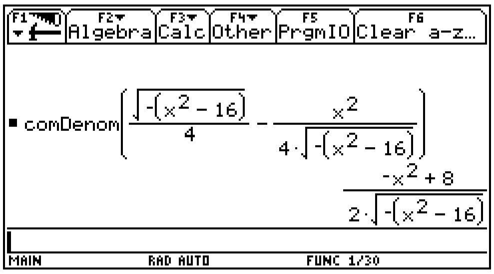
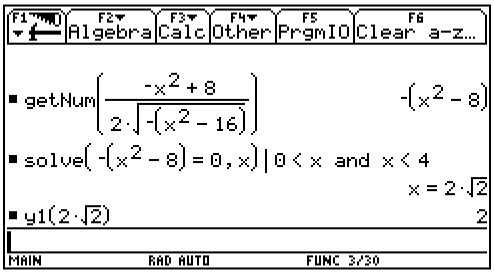
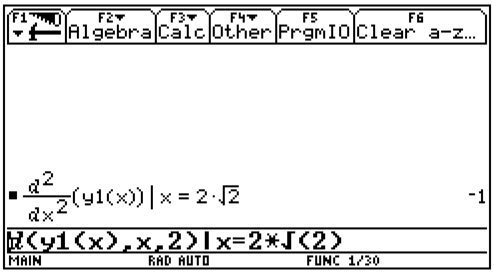








The last screendump illustrates the second-derivative test to prove that the extremum is a maximum in this case.

6. Now we find $A = \dfrac{x.\sqrt{4a^2 - x^2}}{4}$, with a maximum area $\dfrac{a^2}{2}$ when $x = a\sqrt{2}$. Obviously, the maximum area of the isosceles triangle is equal to the length a of the legs only if a = 2.

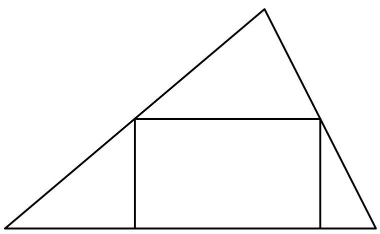
7. The triangle is right angled. This fits with the formula for the area found in 6. $A = \dfrac{a^2.\sin\alpha}{2}$.

8. The maximum area is $\dfrac{a^2}{2}$, when $\sin\alpha$ reaches its maximum value 1 for $\alpha = \pi / 2$. The triangle with maximum area is right angled!

9. Other choices for the independent variable: base/2, base angle, (top angle)/2, height of the triangle.

Example for Student assessment

1. A triangle has vertices with coordinates (0,0), (10,0) and (7,6). Note that this is a triangle with base 10 and height 6.
Prove that the rectangle of maximum area, fitting in the triangle (see figure) has base 5 and height 3.



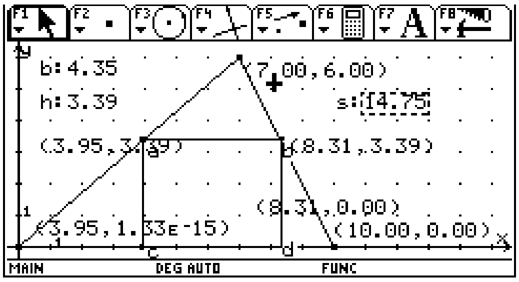
2. We try to generalize: if a triangle has base b and height h, does the rectangle of maximum area, fitting in the triangle, have a base $b/2$ and height $h/2$?
Verify and prove this general statement.

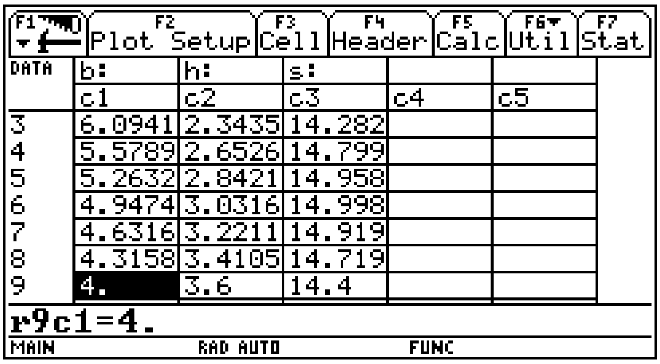
Make a report with a clear explanation of your solution and of the elements you used in your solution process.

Example of Students' Working

It was a remarkable fact that students had problems finding the area of a triangle in terms of two sides and the included angle (see problem 7). Obviously, the presentation of the figure was misleading to them; they took the wrong side (opposite to the given angle) as the base of the triangle.

- In the assesment reports, we observed that students did indeed use different representations.





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4	5.5789	2.6526	14.799		
5	5.2632	2.8421	14.958		
6	4.9474	3.0316	14.998		
7	4.6316	3.2211	14.919		
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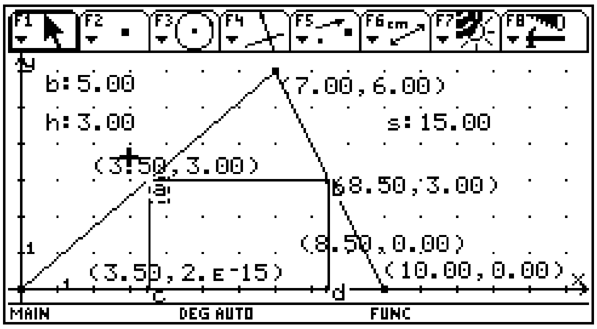

For the first problem some students worked out a CABRI simulation, and used animations and tables to find the rectangle with maximum area.

The generalisation was done analytically. Students made different choices of variable, mixed with geometrical considerations (similarity of triangles). Also, if they led to complicated functions, the use of a symbolic calculator led to correct answers.

Questions and/or Tasks for the teacher

- Producing a dynamic geometrical illustration can take a lot of time. Would you allow the students to do this during the lesson? Looking at different cases, this geometrical illustration can be used to solve the problem here purly through geometry, by looking to the right angle in every case. How can you use this aspect of the first problem in relation to the analytical solution of the problem?

- The same strategy can be used in the assesment example. You can use also CABRI simulations by changing the starting triangle. So you can conjecture that the generalisation must be right. You can use this also to give a proof on a special right angled triangle and use conservational arguments to get a general proof. What do you think about using dynamical geometry as a proof generating tool?
- Tables are not perfect; they only give approximations and you can lose essential information by taking a step which is too large. This is a consequence of sampling the original function in equidistant points to obtain discrete information. What is your opinion about using tables? How would you assess a student's work who has solved an optimization problem only with the table method?
- Change of representation is an important problem solving strategy. Do you think that these examples can be used to teach students this strategy explicitly? Do you think that informatuion and communication technology can play an important role here?
- Discuss the organisation and marking of the assesment example.

References to Literature

Ayres, F., Mendelson, E. (1992). *Differential and Integral Calculus* , 3/ed in SI Units, Schaum's Outline Series, London: McGraw-Hill.

Leinbach, C. (1999): *Using Computer Algebra to Extract Meaning from Parameters.* Chicago: proceedings of the T-cubed Conference.

Luc Gheysens and Guido Herweyers (1999) *Een extremumvraagstuk anders bekeken*, Leuven : T^3-Belgium.

Els Eyckmans (1999) : *Informatietechnologie in het wiskundeonderwijs : een actieonderzoek in verband met afgeleide functie*, licenthiaatsthesis K.U.Leuven, in cooperation with Maria De Rijck (Sint-Jozefinstituut Herentals) and with Christine De Cock (Instituut Berkenboom).

The end of the project

We found a printer to produce 800 copies of our book (300 German and 500 English versions). Tony Watkins´ main part was to polish our English and to produce the camera ready copy of the English version. After overcoming many technical difficulties reaching from compatibility-problems with the zipped textfiles and the graphic files to the contents of an enveloppe with the camera ready papers getting lost we could order printing the 1st issue of our work. The number of German copies was shared between Germany and Austria, the English ones went to the other partners. In a very short time we were "sold out" – to tell the truth we didn´t sell the copies but gave them away on request. The echo, the comments and the reviews were very positive.

The well-known German publisher Klett is now working on a reprint of New Technologies ..... and Dirk Janssens together with Paul Drijvers found a publisher to produce a Dutch translation. It would be fine to have also a French version, but as there was no French partner in our group. Consequently, the aims and the contents of our project were not spread very widely in the French speaking world.

Additionally it should be worth mentioning that T^3-Germany (Teachers Teaching with Technology) accepted the structure of our units for their own teaching materials. I wonder if there are many European Union Comenius Projects which are showing a similar influence on recent developments in education.

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A last word about the project´s financing

The project budget in the application form was about 37610 EURO and we requested a grant about 18500 EURO. We received a grant of 16650 EURO (44.27%). The end calculation showed that we worked very carefully and that we didn´t spend all the money. The project total sum was 25 326 EURO and so we will only get approximately 11200 EURO refunded. We – the PI Lower Austria – sent the end report with cost analysis and waited for acceptance. Unfortunately we received a letter to explain some costs – costs which are exactly the same in the application form. So the project is not really finished.

If anybody is interested in our work in detail then please contact me.: nojo.boehm@pgv.at