

Fourth International Derive TI-89/92 Conference

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Creating Problems for Solution With Technology

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Introduction

With the availability of technology, math assignments can take on a more challenging and interesting style. Many textbooks still contain long lists of “drill and kill” assignments. Often for group projects or for more long-term assignments, teachers must write their own problems that require the student to use technology. This paper suggests several guidelines and ideas to use in writing such problems.

Requiring Technology

The technology that we intend students to use includes graphing calculators, calculators which can do symbolic manipulation, spreadsheets, and even the use of programming languages such as BASIC, Pascal, and C++. The division line between some of these different technologies is getting more and more blurred. All are becoming more available to schools at every economic level. The necessity of technology is dictated by presenting problems with such things as:

- “messy” numbers

- functions that are difficult to graph

- varieties of regression to find an appropriate modeling function

- multiple views of a problem (graphical, numerical, symbolic, textual)

Motivation

Teachers today seek to develop capabilities within their students to be able to think mathematically. Living and succeeding in today’s world requires the ability to visualize mathematical relationships. Graphs and figures come at us from politicians, from advertisers, and from the media. One needs good judgment about mathematical claims made by all these sources. Being able only to factor trinomials or successfully execute the chain rule taking a derivative (which modern calculators will do for us) is not enough. Problems that require thought and analysis and the ability to communicate the interpretation of the results do more to develop the skills needed in today’s world.

Steps for Creating Problems

Sources

In some ways it is daunting to sit down at an empty screen on your word processor and come up with a good problem, starting with nothing. A variety of sources can be used to jog one’s mind and trigger ideas for problems. The World Almanac is a good source for population or finance figures over a period of years or across a country or state. Sports almanacs with records or tables of performances can be useful. Amusement parks present a plethora of applications of physics, linear motion, angular motion, gravitational forces, business strategies, etc. The World Wide Web can be used to find science news reports, graphs and off the wall facts. Interesting quotes can be found in magazines or on web sites which will set one’s mind off in the direction of a problem.

Fourth International Derive TI-89/92 Conference

For example, a headline from the Dayton, Ohio *Daily News* was quoted to say

“Jail Releases Upset Judges”

This could be taken a couple of different ways. However, it suggested a problem on the effects on early release programs and the resulting effects on crime statistics. Would the results be modeled best by linear or exponential functions? What should authorities expect in either case?

One should be careful of lifting questions from existing text books, as copyright issues can be raised. However, they can occasionally be used to give an idea, which could be developed further or in a different direction.

Writing the Questions

The first step is to write a couple of statements that grab the students' interest, for example:

“When driving a vehicle at cruising speed, it costs extra to slow the vehicle to a stop and regain cruising speed again. The cost depends on two variables, both the approach speed and the length of time you are stopped.”

Then present the setting of the problem. At this point a couple of different directions can be taken.

The first possibility is to present in detail the total task – what the givens are and what specific answers or tasks are required. The students are expected to come up with all the intermediate steps on their own. The other direction that can be taken is to carefully lead the students through the derivation of the required formulas and graphs, giving good hints and help each step of the way. In either case, it is sometimes interesting and useful to specify the questions a bit loosely. In real life, problems are often ill defined and students should be given a taste of what it is like to struggle with exactly what it is they are being requested to come up with.

Requiring predictions before a function is derived and/or graphed is a good strategy when writing a question. This should be followed by requesting revision of the prediction or explanation of why the prediction was right. For these kind of textual responses, a full sentence with subject, predicate, and object should be required.

Once functions have been developed, they can be used in a variety of ways. They can be evaluated for the purpose of making a prediction. Functions can be used in combination with or composition with other functions to determine some other value. The result of a function can be given and the value of the independent variable which yields that result can be requested.

Including Variety of Viewpoints

It is important to cover as many mathematical representations of the functions as is possible. Numerical results of a problem can be displayed as tables of values, dimensions of an object, distances, velocities, amount of profit, loss, or sales. Looking at a table of values generated by a calculator can give clues as to the window required to graph the function.

Fourth International Derive TI-89/92 Conference

When one or more functions are graphed the student can look for intercepts, intersections, roots, local maxima and minima, and asymptotes. Often, viewing the results of a graph can suggest other functions which are combinations of what has been graphed. For instance, noticing that two functions intersect, drift apart, and then intersect again, might suggest graphing the difference of the two functions and noting the local maximum or minimum of this difference function to find out where the original two functions are the farthest apart.

Modern technology has the capability to manipulate mathematical symbols and this should be employed by the student as well. Symbolic and functional notation enables mathematicians to prove hypotheses and communicate findings to others.

Along with graphs, pictures and diagrams are important. In some cases these are given to the student to help describe the original problem. The instructor may choose to make the diagram accurate to help or give hints. An opposite strategy is to give a generic figure which either gives no hints or forces the student to be able to generalize the concept.

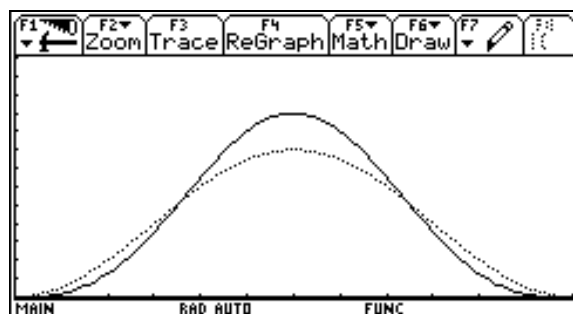
Diagrams may also be required as part of the student's solution. Different people learn different ways and visualization is an important aid to many students.

Finally, students should be required to describe their work in words. This can be done in written and/or verbal format. Full sentences with subjects and predicates should be required. Well-formed paragraphs are also important. Use of visuals (diagrams, formulae, graphs, and tables) should be required. Use of presentation technology should be encouraged.

Kinds of Problems

Technology available today for mathematics provides opportunity for a great host of real world problems. Actual data points can be presented and used as a jumping off point for a variety of questions.

The data points can be plotted and the student asked to suggest one or more types of functions which represent such a curve. Regression capabilities of calculators will then determine the modeling functions. Students can be asked to use the resulting function to give intermediate values within the domain of the original data. They can also be asked to extrapolate and predict values beyond the domain of the data and offer a judgment as to the suitability of the prediction.



Fourth International Derive TI-89/92 Conference

When two functions are graphed on the same axes, as shown above, students can be asked a variety of questions:

- Where are they the same?
- How many times are they the same?
- What might be happening beyond the domain shown?
- Do they have the same x value for their local maxima?
- Is it possible to perform a transformation on one to get the other? If so, how?
- Where is the dotted line function the farthest above the solid line function?
- Where are the slopes of the two functions equal to each other?
- In the context of the problem, what is happening at the intersections, at the zeros, at the intercepts, at the maxima or minima?

Parametric equations and their graphs are quite easy to do with technology. It is possible to model two different objects moving and finding where they meet or cross each other's paths. Students can be asked to derive functions which represent time, rate, and distance for two different objects (say an aircraft carrier and a landing airplane) and by graphing them in simulated time, demonstrate that they do get together.

Levels of Problems

School settings provide a diversity of situations where problems are required. Some problems are required for daily homework. Teachers don't usually write their own homework assignments, depending on the texts to provide these. However a text which assumes little or no technology may need a supplement now and again.

More often instructors will write their own test questions. When technology is available to all the class, some test questions should be written which require that technology. (At the same time, most educators today agree that some skills should be mastered and demonstrated without the use of technology – that's another whole paper!)

Questions or problems which are more lengthy or detailed are suitable for class projects. These projects may be for groups or for individuals. Group problems can be given as a one day, in class exercise to introduce or to apply a current concept of the curriculum. Problems can be given which bring several concepts of the course all together in culmination.

Managing Groups

When group projects are assigned, it is usually better to assign the make up of the groups. Randomizing and then manually altering those results is a favorite tactic of the author. When the groups are to meet during class time, it is also a good idea to give a diagram as to where each group is to congregate.

Structuring all of this ahead of time makes things go smoother and results in less goofing off. Someone in the group (for instance, the first person in the group list or the one who's birthday is closest to that day) should be designated to be the "scribe" who is responsible to record and submit the group's final results.

Fourth International Derive TI-89/92 Conference

Grading

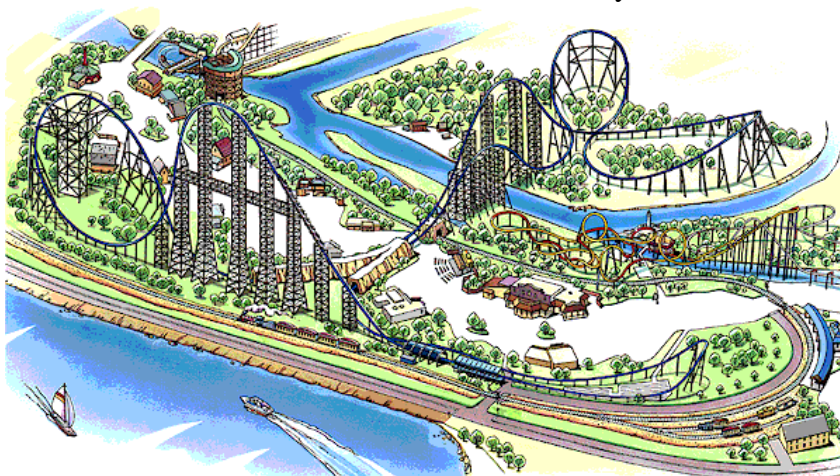
The instructor should read down through the problem presentation handout and note the specific answers for which he/she is looking. Create a checklist noting each of these and assign a value of how many points will be awarded/deducted as they are evaluated. Use of a checklist keeps the grader (instructor or student helper) objective. It also saves the grader from writing the same note on multiple papers. This checklist can be given to the class ahead of time (when the assignment is made) or not (assuming that they should be reading the problem carefully and determining on their own what must be included).

Students should be informed ahead of time as to how the grade is to be counted. Is it a homework weighted grade, equal to a test, a quiz, or even percentage points on an upcoming test. When the problem is worked on in groups, the group with the highest score and earliest submission can be awarded bonus points.

Demonstration

To apply the principles being presented, we will now organize our meeting into groups and work together on writing a problem. The problem will be based on the following scenario:

Roller-coaster enthusiasts often argue over which is the best car of a train in which to ride. Some say the front seats are more exciting because you see better. Others say that the back seats (100 ft behind the front seat) are going faster on dips and curves and thus pull more G's. The new Millennium Force Roller-Coaster at Cedar Point Amusement Park in Sandusky, Ohio, has a first hill 310 feet tall that goes



down at an angle of 80° . The cars will hit a maximum of 92 mph (134.9 ft per sec). The length of the track is 6595 feet and takes 2 minutes and 45 seconds for the entire ride.

Now ... write a problem that requires technology for its solution ...

Your group will write a problem using the information given. Your problem must use technology for its solution. Try to use all of symbolic, numeric, graphic, and textual representations in presenting and/or solving the problem. For this exercise, lead the students through the steps required (help them

Fourth International Derive TI-89/92 Conference

derive the formulas, etc.) to arrive at the final objective. Make sure that the final objective is specifically and clearly stated. Suggest additional or extended searching that they might pursue for enrichment. The level of the problem should be specified to be for algebra, pre-calculus, or calculus. The person in your group who has the first letter of your family name closest to the end of the alphabet is the scribe of your group and will write up your problem for presentation to the rest of us.

Technology Problem Writing Evaluation

| Requirement | Points Off | Possible |
|--|------------|----------|
| 1. Technology used for solution | _____ | 15 |
| 2. Symbolic representation used | _____ | 8 |
| 3. Graphical representation used | _____ | 8 |
| 4. Numeric representation used | _____ | 8 |
| 5. Textual representation used | _____ | 8 |
| 6. Lead students through development of formulas | _____ | 5 |
| 7. Final objective clearly stated | _____ | 5 |
| 8. Level of problem specified | _____ | 5 |
| 9. Other | _____ | |
| _____ | | |
| Total points = 62 – total points off _____ = _____ | | |

Name of Group : _____

We will form groups and spend 20 minutes or so with each group creating a problem as specified. If time permits, the groups could then present their work. I am trying to model some of the things I have advocated (group work and evaluation) as well as give conferees an opportunity to apply the concepts that have been presented.