Contents

Fourth International *DERIVE*-TI-89/92 Conference
Liverpool John Moores University, 12-15 July 2000

Using the TI-92 Plus: Some Examples

Michel Beaudin
École de technologie supérieure,Canada
mbeaudin@seg.etsmtl.ca

1. Introduction

We incorporated the use of the TI-92+/89 in order to help our engineering students use Computer Algebra in the classroom, not only in the labs (since September 1999, every new student has to buy a TI-92+ or a TI-89). Students are often asked to use the « rule of four » : where appropriate, topics should be presented geometrically, numerically, analytically and verbally. With a tool like the TI-92 Plus, we can present and perform examples that will use that rule. We will present some examples from our first calculus course, some from our multiple variable course and some from our ordinary differential equations course.

2. Examples in single variable calculus

2a) Example : equation solving (graphically, « solve », « nsolve », Newton's method)

Students learn that every exponential growth function eventually dominates every power function. Let's see what is going on when we want to solve the equation

$$3^x = x^{16}.$$

There is no formula for the solutions of the above equation (and there is no need of « Lambert-W function » in a first calculus course !). A plot, in the same window, of the sides shows two intersections and the « solve » command gives a warning that there might be more solutions (figure 1) !

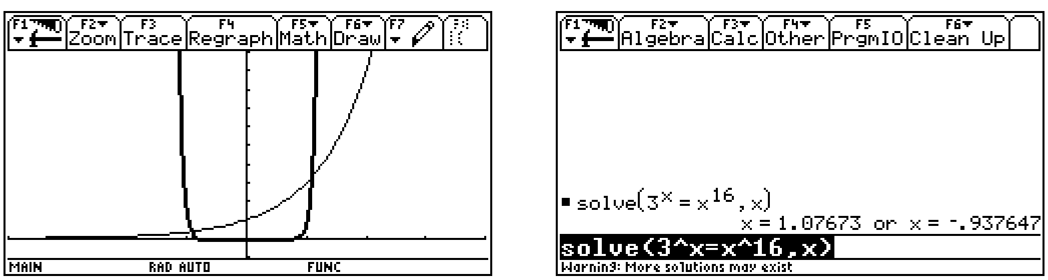


FIGURE 1

But there is, in fact, a third intersection, because every exponential eventually dominates every power! A table shows that it is between 50 and 60 and using « nsolve » with appropriate guess allows us to find this third intersection (figure 2 on the right).

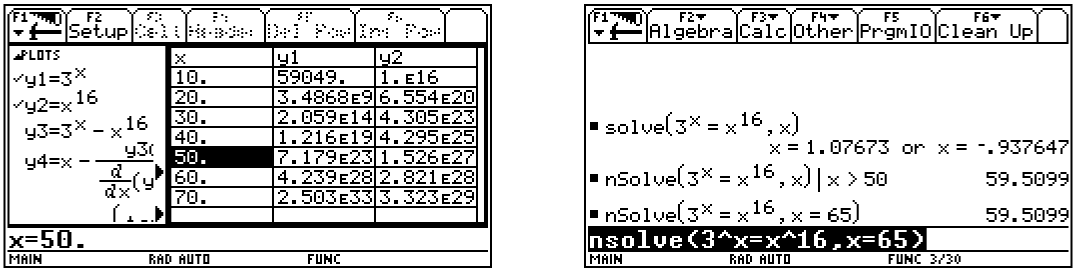


FIGURE 2

Because Newton’s method is the one used by students as an application of the derivative, we try to find the first positive intersection (figure 3) simply by asking the TI to perform the iteration

$$x - \frac{f(x)}{f'(x)} \quad \text{where} \quad f(x) = 3^x - x^{16}$$

This can be done in the Y-Editor : no need to program a « Newton » function !

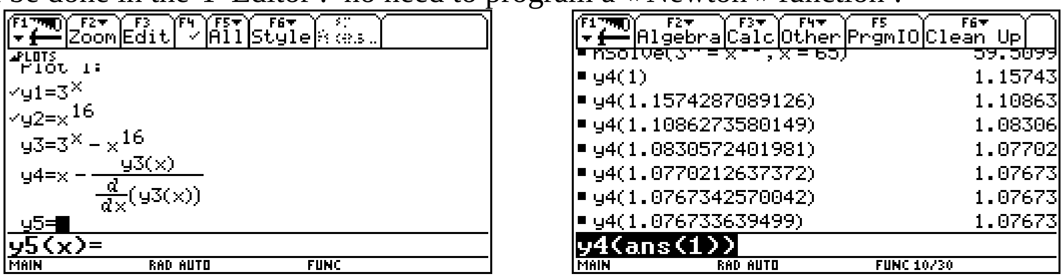


FIGURE 3

As we can see, applying Newton’s method is fast and quite easy with the TI ! Finally, for positive *x*, our equation is equivalent to

$$\frac{\ln x}{x} = \frac{\ln 3}{16}$$

and now, a graph shows that there are exactly two positive solutions for our equation : the TI helps the students to remember properties of logarithms.(figure 4 on the left) !

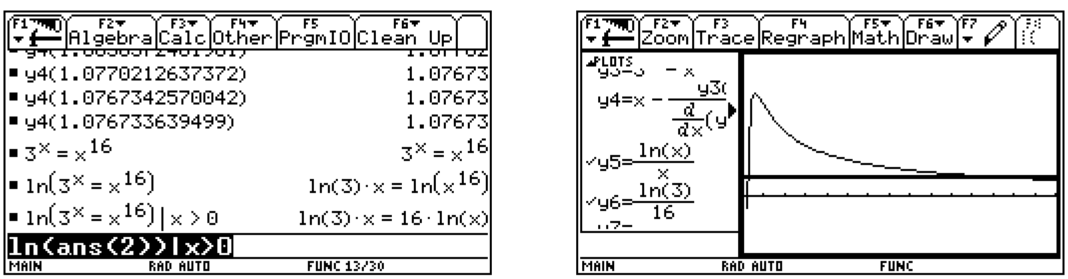


FIGURE 4

2b) Example : finding limits in many ways.

The concept of limit is best understood by students if we use the rule of four again. Suppose we want to make a conjecture about the value of

$$\lim_{x\to 0}\frac{2\cos(3x)+9x^2-2}{2x^3}$$

(this problem is encountered before studying l'Hopital's rule). We are going to make a table of values of the expression for $x = \pm 0.1, \pm 0.01, \pm 0.001$, (figure 5 on the left) as well as we will look at the graph of the expression (the TI shows the « whole » at the origin !)

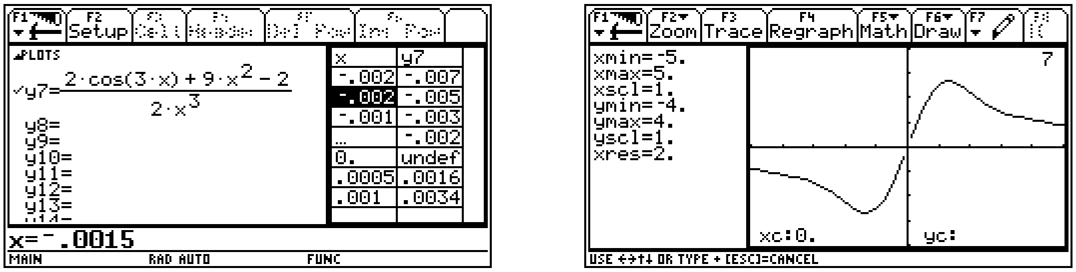


FIGURE 5

Now we will find a value of δ that will work for a value of ε, say ε = 0.01 : this means, that we will find an interval containing 0 such that the difference between our conjecture and the value of the expression is less than 0.01 on that interval (we will have to zoom in order to find a window of height 0.02 such that the graph exits the sides of the window and not the top or bottom of the window). We set the window parameters for *y* between –0.01and 0.01 and try for δ the value 0.05 (so we fix *x* between –0.05 and 0.05). Figure 6 (on the left) shows that the value of δ is not small enough ! Zooming in for *x* only yields a corresponding value of δ (figure 6 on the right), in fact 0.0025 seems to work for δ but the graph shows that we need to find this limit analytically !

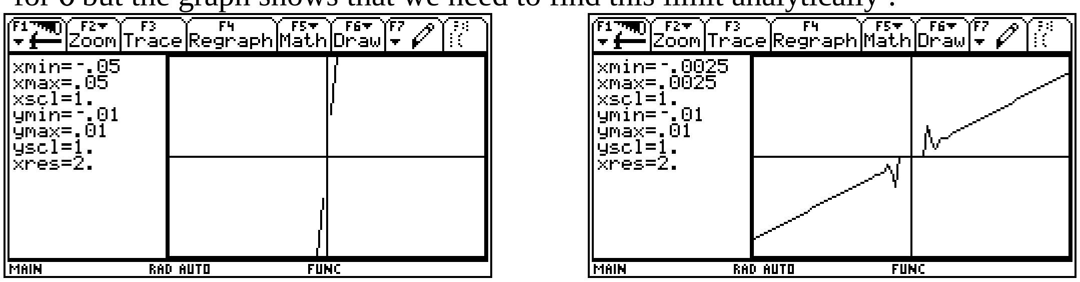


FIGURE 6

Of course, l'Hopital's rule or a Taylor expansion of the numerator's expression would be the « analytic » part of the « rule of four » (figure 7).

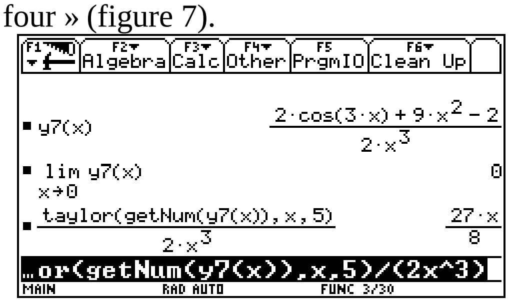


FIGURE 7

Contents

2c) Example : collision or just an intersection ?

Here is a funny example that leads to a better understanding of 2D-parametric curves. Suppose we have two particules C_1 and C_2 moving in the same plane, their respective trajectories described by the parametric equation

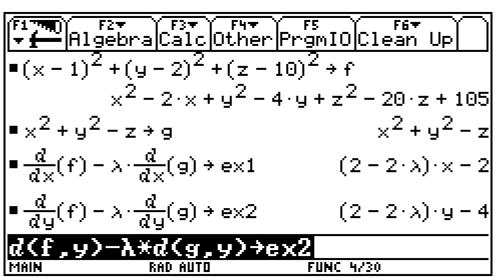
$$C_1 : \begin{cases} x = \dfrac{t^2}{8} - \sin t \\ y = t - 3\cos 2t \end{cases} \qquad C_2 : \begin{cases} x = \sin\left(\dfrac{t}{2} + 1\right) \\ y = \dfrac{2}{t^2 + 1} \end{cases}$$

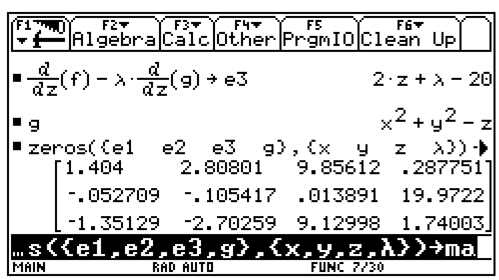
each for $0 \le t \le 5$. Are the trajectories crossing ? If so, is there a collision and at which moment is this happening ? I suggest you ask your students these questions ! They will find that, when trying to solve a non-linear/non-polynomial system, it is a good idea to give a starting point, for example the one given by the graph.

3. Examples in multiple variable calculus

3a) Example : using or not the Lagrange multipliers.

Find the minimum distance from the point (1, 2, 10) to the paraboloid given by the equation $z = x^2 + y^2$. It is fun to solve this problem with and wihout Lagrange multiplier ! If we minimize the square of the distance, we have to minimize the function $f = (x-1)^2 + (y-2)^2 + (z-10)^2$ under the constraint $g = x^2 + y^2 - z = 0$. That means to solve the non linear system $\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$. Solving the first three equations for x, y and z and combining with the fourth one yields the cubic equation $\dfrac{5}{(1-\lambda)^2} = \dfrac{20-\lambda}{2}$. This former equation has 3 real roots and we find that the minimum distance is 0.9148, at the point (1.4040, 2.8080, 9.8861). The following screens show complete solution of this problem. We can note the use of the « zeros » function of the TI-92+.





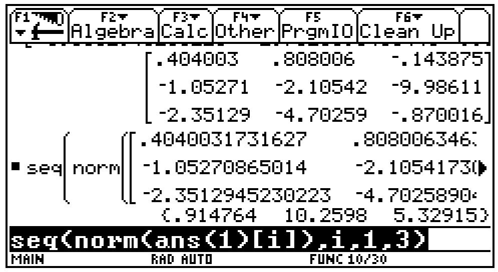


FIGURE 8

Another approach will be to substitute the constraint into *f* and to consider the function that yields the distance from the point (1, 2, 10) to an arbitrary point on the paraboloid :

$$\sqrt{(x-1)^2+(y-2)^2+\left(x^2+y^2-10\right)^2}$$

Here is the graph of the last expression :

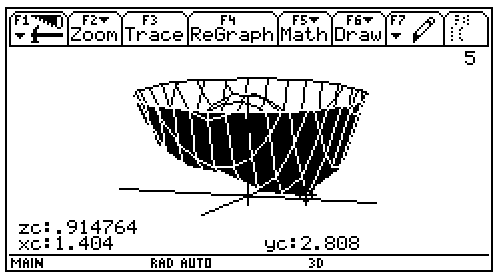


FIGURE 9

3b) Example: the user still have to understand what is going on ! ! !

Consider, for example, the following double integral $8\int_0^1\int_0^{\sqrt{1-x^2}}\sqrt{1-x^2-y^2}\,dy dx$. It is, of course, the volume of the unit sphere, so the correct answer is $\dfrac{4\pi}{3}$. When we want to perform the integration, we use polar coordinates : see figure 10.

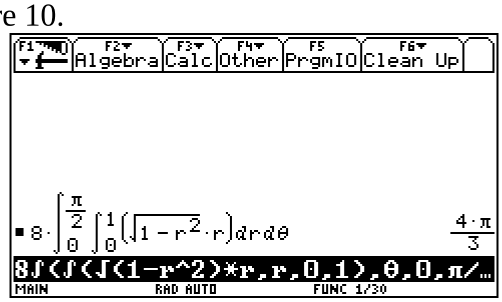


FIGURE 10

If we ask the TI to perform the calculations using rectangular coordinates, this happens :

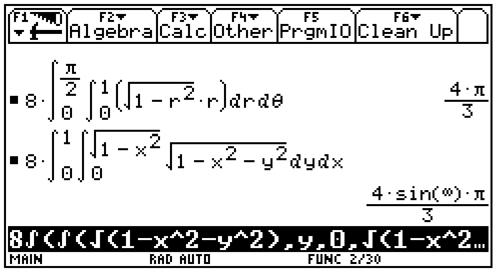


FIGURE 11

Because « sin (∝) » is something between –1 and 1, the answer is correct ... (DERIVE 5 gives the correct answer : but try it and simplify the inner integral ...).

4. Examples in differential equations

4a) Example : a slope field is important ! ! !

In differential equations, students solve ODEs without looking at the slope field and without finding where the solution is defined ! Let's consider, for example, the first order separable equation

$$\frac{dy}{dt}=\frac{3t^2+4t+2}{2(y-1)},\ y(0)=-2\,.$$

Of course, it is easy to solve by hand this DE and we find $y^2-2y-8=t^3+2t^2+2t$. If we want to solve for *y*, there will be two answers, depending on wich branch we want to select (because *y* = 1 is a singularity and the slope field allowed us to see this). If we apply Euler's method starting at *t* = 0 and *y* = –2, with a negative step, we will encounter problems ... (figure 10 on the right ; note that the RK method used by the TI-92+/89 « recognizes» that there will be a problem and stops drawing when we approach *y* = 1 : figure 12 on the left)

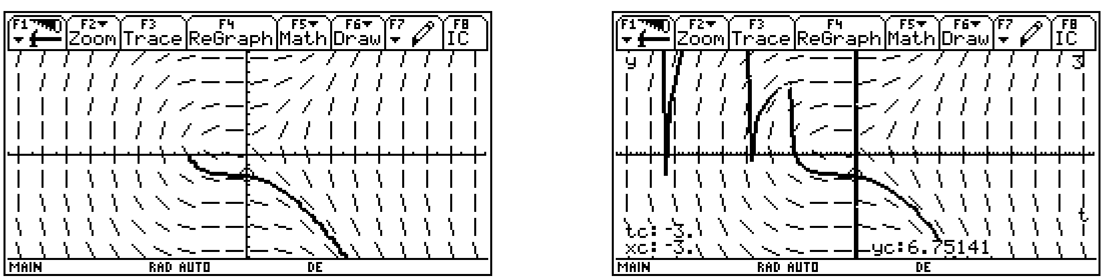


FIGURE 12

4b) Example : a linear first order ODE that leads to the concept of steady-state solution. Let us consider, for *a* a real number, the following differential equation

$$\frac{dy}{dt}+y=3\cos(2t),\ y(0)=a$$

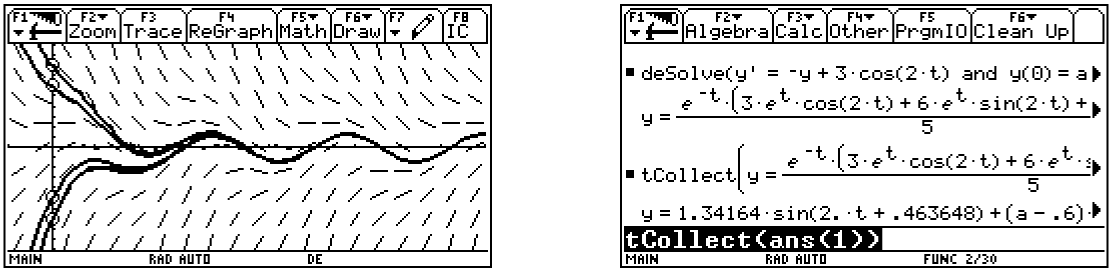


FIGURE 13

The interactive IC menu F8 allows us to choose the initial conditions and a solution is drawn by RK or Euler methods. (figure 13 on the left). This can be done before solving exactly the first order linear equation $\frac{dy}{dt} + p(t)y = q(t),\ y(t_0) = y_0$. And, using the « deSolve » function, we see easily that the answer is the sum of a transient solution and a steady-state solution whose amplitude seems to be 1.34164 and phase angle 0. 433648 using, for example, Trigcollect (figure 13 on the right).

4c) Example : a direction field and more for two interacting populations (predator-prey).

After some analysis, suppose we have to solve the first order following system :

$$\begin{cases} \dfrac{dw}{dt} = w - wr \\ \dfrac{dr}{dt} = -r + wr \end{cases}$$

A very good analysis is now possible with the TI. The following screens (Figures 14a amd 14b below) show all the results. The direction field with a solution curve (closed curve) starting at the point (3, 2) was drawn after choosing DIRFLD and RK ($y3$ stands for w and $y4$ stands for r). Periodicity is now clear from graphing population as functions of time. Finally, how good is RK ? Well, if we solve the separable ODE

$$\frac{dr}{dw} = \frac{-r + wr}{w - wr},$$

we get an implicit solution, that can be drawn with the aid of the 3D implicit plotter and pasted to the curve given by RK (we switched to the « line » style for RK in the last screen below in figure 14). As we can see, the implicit plot was not so good.

Contents

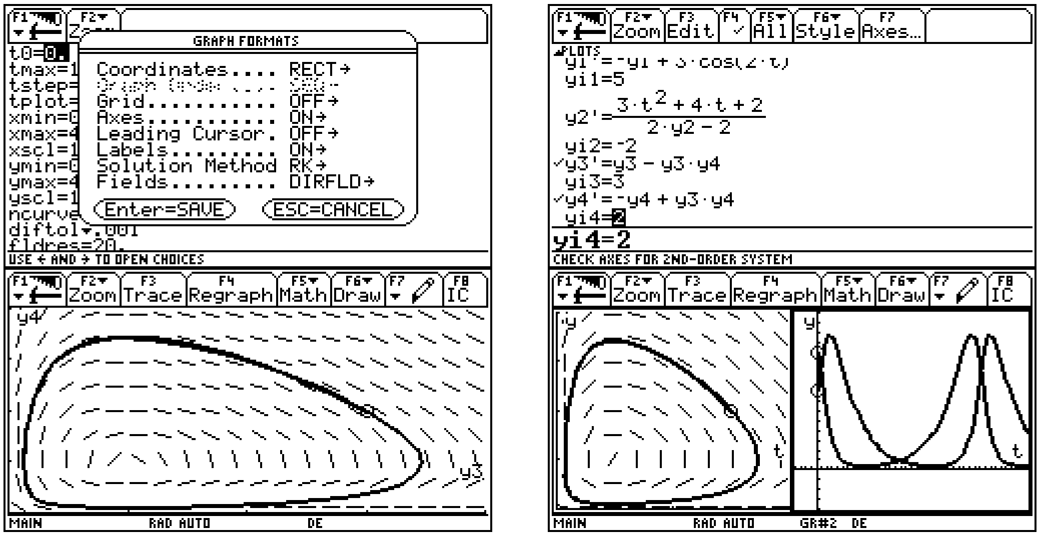


FIGURE 14a

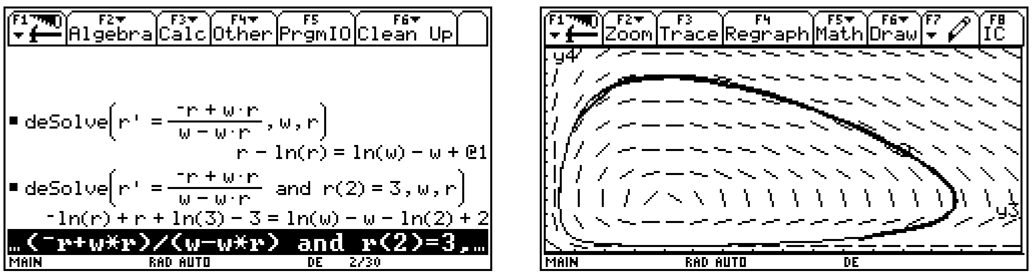
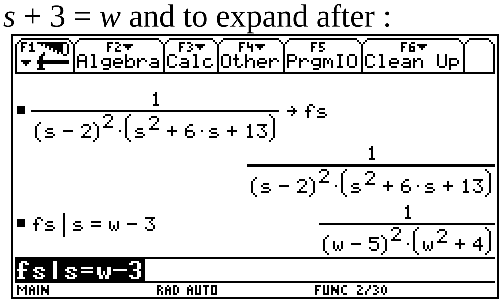


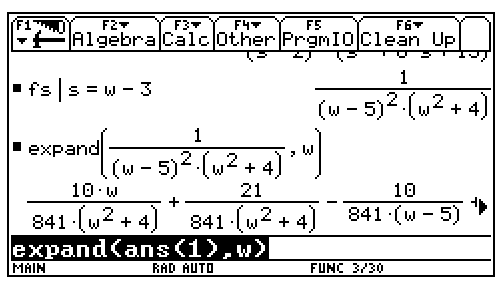
FIGURE 14b

5. Examples for Laplace transforms/Fourier series

5a) Example : many ways to find an inverse Laplace transform !

Finding the inverse Laplace transform of an expression using partial fraction and/or convolution ! Suppose we have a table of Laplace transforms and we want to find the inverse of the transform $F(s) = \dfrac{1}{(s-2)^2\left(s^2+6s+13\right)}$. Because a completion of the square has to be done, it is better first to set $s + 3 = w$ and to expand after :





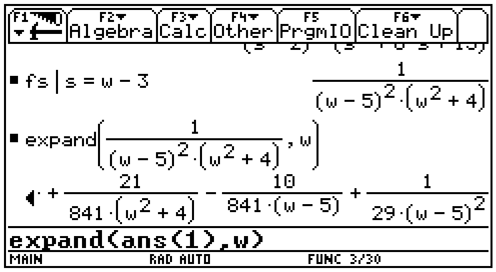


FIGURE 15

A table of transforms contains the following associations : $\dfrac{s+a}{(s+a)^2+b^2} \leftrightarrow e^{-at}\cos bt$, $\dfrac{b}{(s+a)^2+b^2} \leftrightarrow e^{-at}\sin bt$, $\dfrac{1}{s+a} \leftrightarrow e^{-at}$ and $\dfrac{1}{(s+a)^2} \leftrightarrow te^{-at}$. So, the inverse is simply (because $w = s + 3$)

$$\frac{10e^{-3t}\cos 2t}{841}+\frac{21e^{-3t}\sin 2t}{1682}-\frac{10e^{2t}}{841}+\frac{te^{2t}}{29}$$

Another approach our students are encouraged to use is the convolution. We have

$$F(s)=\frac{1}{(s-2)^2\left(s^2+6s+13\right)}=\left(\frac{1}{(s-2)^2}\right)\left(\frac{1}{(s+3)^2+4}\right)\leftrightarrow te^{2t}*\frac{1}{2}e^{-3t}\sin 2t$$

where * means the convolution of signals. The TI can do it for us if we simply use the definition of the convolution : $x(t)*h(t)\equiv\displaystyle\int_0^t x(\tau)h(t-\tau)\,d\tau$.

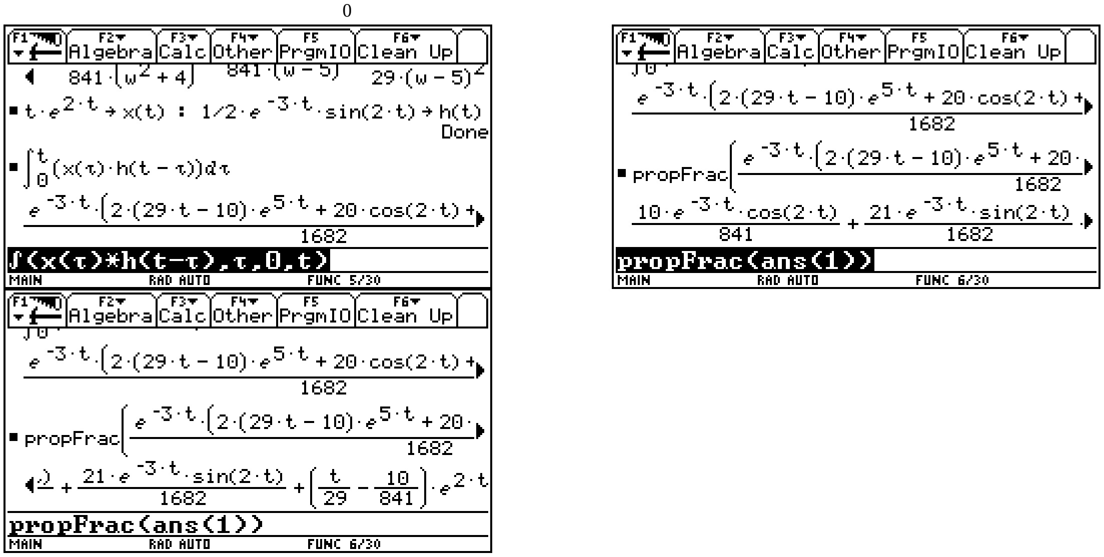


FIGURE 16

5b) Example : Fourier series and the energy theorem.

It is very simple to calculate a Fourier expansion with a TI-92 ! Let f be the periodic function of period $P = 3$, defined by

$$f(x)=\begin{cases} x \text{ if } 0<x<2 \\ 2 \text{ if } 2<x<3 \end{cases} \quad P=3$$

We define this function using the « when » and extend it periodically by using the modulo function (same as in *DERIVE*). We note that the TI connects the discontinuity (figure 17 on the right) :

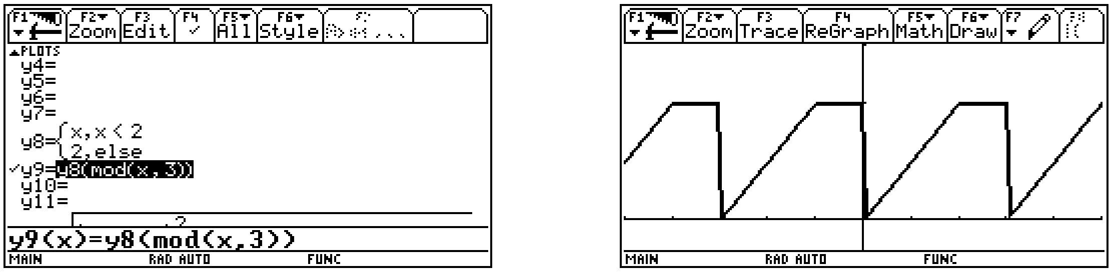


FIGURE 17

The TI knows that a number n is an integer if we use for example the arbitrary integer $@n1$. We calculate the Fourier coefficients :

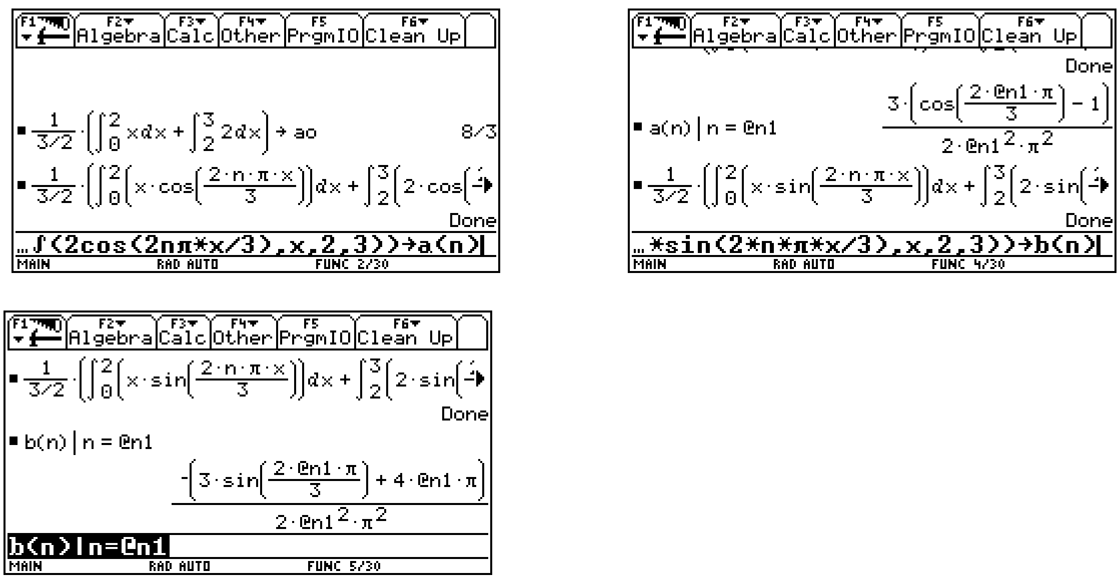


FIGURE 18

We will plot the partial sum $\dfrac{a_0}{2}+\sum_{n=1}^{3} a_n \cos\left(\dfrac{2n\pi x}{3}\right)+b_n \sin\left(\dfrac{2n\pi x}{3}\right)$ and the periodic signal in the same window (figure 19 on the right).

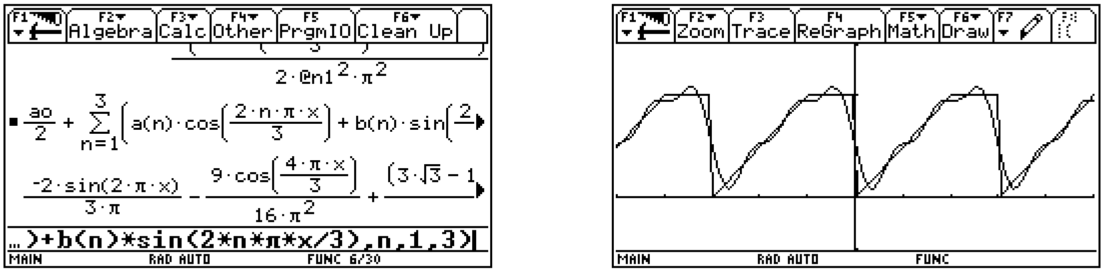


FIGURE 19

We conclude this example and this paper by looking at the Energy Theorem (Parseval identity) that states that

$$\frac{2}{P}\int_P f^2(x)\ dx = \frac{a_0^{\ 2}}{2} + \sum_{n=1}^{\infty} a_n^{\ 2} + b_n^{\ 2}$$

What fraction of the energy of *f* is contained in the constant term and the first three harmonics ? About 97.3677% :

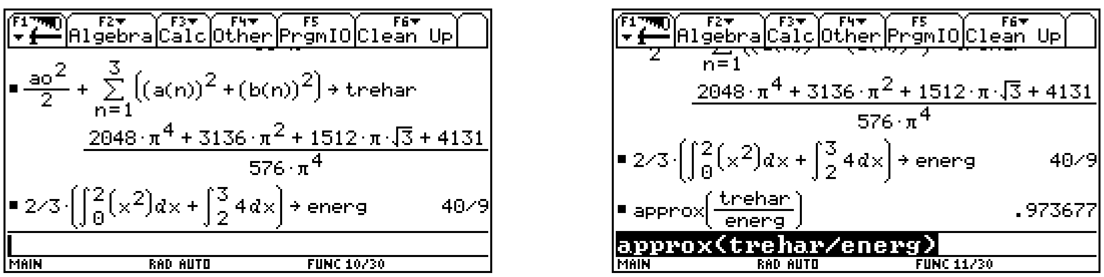


FIGURE 20

6- Conclusion

Teaching mathematics with a CAS like *DERIVE* has been a great experience for us since the last 5 years. Being able to see our students in the classroom performing calculations and examples like those presented in the precedent section, on their own « portable CAS », is fantastic. Thanks to those who worked so hard in order to « take out » computer algebra from big computers and labs! Long life to these machines ! I hope that the examples presented in this paper will convince those who think that the consequence of using a portable CAS will be less mathematics in our teaching. We will continue to teach traditional mathematics but we will be able to perform calculations that were impossible before the introduction of these portable CAS.