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Some Reflections on the Uses of Computer Algebra in Teaching, Learning and Assessment

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Abstract

For about a decade, the author has been using computer algebra packages in a wide variety of teaching, learning and assessment situations. Courses taught include first year service modules offered to scientists whose entry qualification is generally no more than a partially forgotten GCSE obtained with a moderate grade, students whose main areas of interest include computing and accounting, engineering students who have been offered computer algebra courses on a voluntary basis and students whose main interest includes mathematics, normally allied to another subject of their choice. The author has used DERIVE and/or Maple with many of them on a variety of appropriate platforms including SUN and Macintosh (for Maple) and PC (for DERIVE and Maple). In most cases, the package used has been viewed as an integral part of the courses offered rather than an "add-on" simply to be used during tutorials. This philosophy has resulted in the author using a wide variety of computer algebra based assessment methods including group work using both self and peer assessment as methods of involving students in the assessment process and "standard" three-hour examinations for which an algebra package is available with the examination being sat at the computer.

In many cases, classes have involved the use of specially written learning material supported by regular classroom demonstrations with the students encouraged to participate interactively immediately prior to attempting set tutorial work on their own or in organised self-help groups. Student reaction to this heavy use of the computer in mathematics classes is discussed.

The author will trace his personal journey through the educational experiences outlined above and openly discuss both successes and failures. Some implications for classroom teaching, organisation, and assessment methods are discussed. The paper will conclude by presenting a set of guidelines recently used by the author to ensure that the use of computer-based examinations is as successful and trouble free as possible.

1. Introduction

One of the problems permanently facing university staff responsible for the service mathematics components of many degree schemes is that in the drive to enable students to cope with the analytical parts of the programmes of study, mathematical demands are made on them for which they are unprepared by school mathematics. Up to GCSE level, students appear to be able to gain, say, a grade "C" with a very sparse knowledge of algebra and in particular a very low level of manipulative ability. In the experience of the author, many such students are allowed, indeed encouraged, to register for programmes including a range of sciences and engineering. This trend appears set to continue into the future fuelled by the wish of the government to widen access to

Higher Education and the lack of well-qualified scientists and engineers. Even some ten years ago, the problems created by the lack of mathematical qualification, drive and ambition in many science and engineering students was giving university staff considerable cause for concern. During the academic year 1991-92, the author sought, along with colleagues, to urgently review the content, delivery and assessment of service modules in mathematics offered to students at Sunderland.

It was decided that the review must pay due attention to the learning, teaching and assessment methods used in the delivery of the courses in mathematics as well as the content of those courses. It should be remembered that the content of the type of service modules under consideration here is determined to a large extent by the Schools receiving the service. Programme leaders have a duty to ensure that service module content will prepare the student adequately for the more analytical aspects of their studies and at Sunderland it is not uncommon for mathematics staff to negotiate content with them. However, such constraints need not, in the view of the author, prevent experimentation either in the modes of delivery used, the variety of learning experiences offered, or the assessment methods used. The reasons for the changes introduced in service modules aimed at students with GCSE qualifications during the period under consideration may be summarised as follows:

- many of the existing service courses, both mathematics and statistics, provided for science students consisted almost entirely of traditional lectures given to classes over 200 strong. These lectures were supported by traditional problem solving tutorial sessions where attendance was voluntary. Attendance at both lectures and (particularly) tutorials was giving cause for concern;
- the assessment methods used were almost entirely dependant on the performances of the students in traditional examinations although some work was done using time constrained tests in multiple choice format to facilitate rapid marking and feedback to students. Failure rates were increasing as were complaints and a general feeling of student dissatisfaction had begun to permeate classes;
- all of the service courses offered ignored the potential impact on learning, teaching and assessment of modern microcomputer software;
- the traditional teaching methods employed in the context of large classes tended to reinforced the jaundiced view of many students that mathematics is a dull, fossilised subject, taught using chalk and talk, done using pencil and paper with little or no relevance to the world in general and their programme of study in particular;
- the courses offered, by their design and scope and delivery, mitigated against the integration of computer-based approaches to learning, teaching and assessment.

2. Rationale

Over the last fifteen to twenty years, university scientists and engineers have become increasingly critical of the mathematics and statistics courses offered to their students by staff from specialist departments. For their part, many university mathematicians and statisticians have felt compelled to reply to the comments concerning the courses they offer by making critical remarks about the quality and qualifications of the students they were expected to teach. The resulting situation, sometimes involving something little short of retrenchment by both parties has done little to address the basic problems faced by students or to develop positive working relationships between mathematicians and engineers or scientists. As a result, and in spite of many well-meaning curriculum revisions in mathematics service programmes, a perceptible drop in the content and level of the mathematics taught by specialists has occurred in many universities. This drop has resulted in an increasingly large body of highly relevant mathematics either not being taught or being allegedly delivered by the host department as part of its specialist discipline. The claim that "we teach the mathematics as part of our modules when we need it" is now commonly made by both engineers and scientists. The author challenges this claim for the following reasons:

- firstly, it is now common to see a reduction in both the number and length of the mathematics service modules incorporated in newly validated versions of engineering and science courses;
- secondly, it has become common in recent academic years to hear of the closure of mathematics mainstream courses and the consequent closure or diminution of the associated departments thus leaving a smaller core of mathematicians employed to deliver service courses;
- thirdly, in many post-1992 universities and some older institutions, the placing of a wide variety of courses on an imminent closure list has resulted in some ring-fencing by departments with staff encouraged or allowed to increase their teaching hours in order to avoid retraining, or the very real threat of redundancy. Regretfully, it appears that mathematics departments can do little to counter this;
- fourthly, and crucially, it is the authors experience as a member of several recent validation panels concerned with both science and engineering programmes that the mainstream modules offered do not explicitly contain a mathematics syllabus nor are relevant teaching materials immediately available for inspection.

On a more positive note, much effort has generally been made to help a wide spectrum of students, especially through the provision of:

- mathematics help schemes which are generally available to all students in an institution;
- diagnostic testing, either paper-based or computer-based, although it is acknowledged that the additional lectures and/or tutorial sessions arising from such testing do add to the burden of students who are already struggling;
- computer-based self-help material such as CALMAT or *mathwise*. While the author believes that these packages do have a positive role to play in modern mathematical education, the comments made above concerning the added burdens experienced by students also applies in this situation;
- mathematics courses designed to involved relatively little formal lecturing but a large proportion of tutorial work involving modern computer-based mathematical tools such as Maple, *Mathematica* or DERIVE.

The use of algebra packages such as these implies a revision of service curricula to include a more student-centred approach, a commitment to deliver mathematics in a more exciting and personally involving way and to take advantage of these tools to aid computation, exploration and assessment. It appears reasonable to suggest that courses which attempt to centrally involved computer algebra in their delivery should also address the problems inherent in using computer algebra in their assessment.

Implementing the Courses

Staff of the School of Computing, Engineering and Technology at Sunderland have used computer algebra systems (usually DERIVE and Maple) as an integral part of the delivery and assessment of mathematics courses for about a decade. While these courses have ranged from first year service mathematics to postgraduate mathematics, comment here will be restricted to first year mathematics involving mainly, but not exclusively DERIVE. Four developments will be of interest:

- a short DERIVE-based mathematics course offered on a voluntary basis to first year engineers;
- a short Maple-based mathematics course offered on a voluntary basis to second year engineers;
- a combined mathematics and statistics course taught as a core module (20 credits) to first year scientists;
- a full module (20 credits) taught to students registered on the University Combined Programme intending to continue with mathematics after their first year.

In the first and second cases, the students taught generally had a respectable level of mathematical achievement (many held an A-level pass) and, in most cases little antipathy towards the subject. In the third case, the students generally held a partially forgotten grade “C” GCSE pass in mathematics but had to attend a mathematics/statistics course. Until recently they were offered a choice of two, the one mentioned here and one which was taught traditionally.

3.1 First and Second Year Engineering Courses

At the University of Sunderland, engineering mathematics courses have never been delivered using the computer as an integrated tool, but the author believes that all students of mathematics, “service” or “main” should have a working knowledge of at least one algebra system as part of the process of their acquiring transferable skills. With this aim in mind, the author has offered extra curricular courses using DERIVE to students in their first year and Maple to students in their second year. Generally, students have been taught in groups of twelve, this number reflecting both the authors wishes to ensure adequate personal contact with each student and the limit (at the time) of the departments DERIVE related resources. Neither course is assessed and while the take up rate has not been specifically monitored it is in the region of 50-60% of the engineering student body. At level one, DERIVE was introduced to students and examples presented whose aim was to enable the student to gain some familiarity with the scope and limitations of the package. Traditional engineering mathematics example sheets, provided by those responsible for the formal delivery of first year engineering mathematics at Sunderland, were used to demonstrate power and ease of use of the package and to raise student interest in the application of computer technology to the processes of learning and doing mathematics. At second year level, the author was, until recently, responsible for both the formal delivery of the engineering mathematics offered and for the introduction of a complementary Maple-based computer algebra course. Both the formal course and the complementary computer algebra course were restricted to the study of Fourier Series and Laplace Transforms. Advantage was taken of the availability of the computer to implement areas of these topics which appear to give students considerable problems, for example:

- the introduction of Lanczos Sigma Factors, which, following a suitable derivation, enable students to compare the graphical output arising from partial sums obtained from the “standard” series

$$f(x) \cong a_0 + \sum_{k=1}^{n} \{a_k \cos(kx) + b_k \sin(kx)\}$$

and the "smoothed" series

$$f(x) \cong a_0 + \sum_{k=1}^{n} \frac{\sin(k\pi/n)}{k\pi/n}(a_k \cos(kx) + b_k \sin(kx));$$

- the solution of systems of ordinary differential equations with a corresponding investigation into the graphical representation of the solutions. A typical "idealised" model in which damping has been omitted for simplicity, both masses are assumed to be unit masses and all three springs have the same modulus is shown below in Figure 1. Vibration occurs between two fixed point *A* and *B*.

The equations of motion of the system are

$$\frac{d^2y_1}{dt^2} = -ky_1 + k(y_2 - y_1)$$

and

$$\frac{d^2y_2}{dt^2} = -ky_2 - k(y_2 - y_1)$$

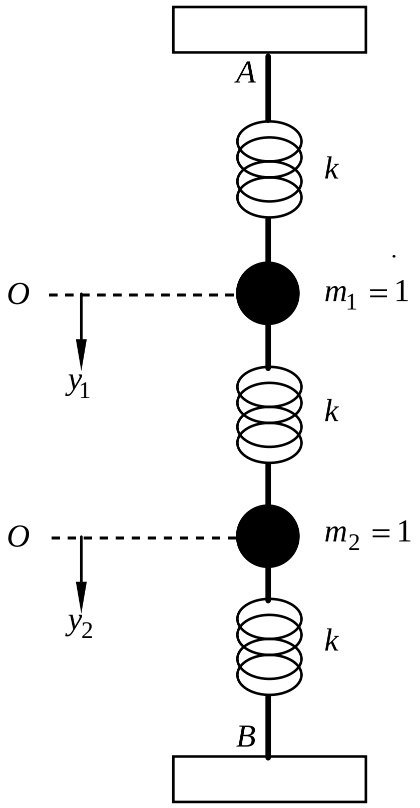

Figure 1 – Undamped Vibrating Masses

Students are asked to justify the equations of motion and implement Laplace transform methods using Maple to find the solution of the system subject to a given set of initial conditions, say, $y_1(0)=1, y_2(0)=1, y_1'(0)=\sqrt{3k}, y_2'(0)=-\sqrt{3k}$. On achieving the solution they are asked to choose a suitable value of k and plot the solutions on the same axes in order to compare them.

3.2 Courses for First Year Science Students

The particular course referred to here ran only for some three years. Course content was largely traditional for a first year mathematics module aimed at the vast majority of students who had no mathematics beyond GCSE. Basic algebra, logarithms, trigonometry, elementary calculus and an introduction to matrix algebra formed the mathematical diet. Traditional hour long lectures were replaced by laboratory sessions which were used to:

- introduce new topics in mini-lecture format using specially prepared hand-outs;
- illustrate mathematical concepts using computer-based or traditional methods as appropriate;
- give the students weekly targets which they were expected to meet;
- present exercises - some open ended - for the students to try;
- give the students experience of group working and encourage the development of transferable skills such as worth assessment, intellectual honesty and the ability to fulfil a variety of roles within a group learning structure;
- ask students to accept responsibility for a range of activities from the management of learning and personnel management within their group to the construction and presentation and of a report;
- ask students to play a part in the assessment of their group efforts by incorporating elements of self and peer assessment in the final grading.

Some experiences gained by the author when using this assessment paradigm are discussed in Section 4 below.

3.3 Courses within the University Combined Programme

This group of students normally consists of those who wish to continue to study mathematics after their first year by taking one of three options:

- the Minor option - roughly one third of the students' time is spent on the mathematical sciences;
- the Dual option- roughly one half of the students' time is spent on the mathematical sciences;
- the Major option- roughly two thirds of the students' time is spent on the mathematical sciences. Students opting for the Major route are required to submit a project in an approved aspect of the mathematical sciences;

As one might expect, the first year syllabus contains a considerable amount of single-variable calculus and linear algebra. Assessment involves a "standard" three hour examination (50%) and coursework (50%) which has several components normally consisting of:

- a time constrained test;
- an open learning assignment;
- an open ended investigation;
- set problems.

While many of the students hold an A-level pass in mathematics, this is not true of all of them, some hold an AS-level pass, some are allowed to register on the Combined Programme as a result of the successfully study of an access course and others are mature students who may hold mathematical qualifications but have not formally studied for some time. Until recently, two computer algebra packages were used with these students, DERIVE and Maple. Currently only Maple is used since the School of Computing, Engineering and Technology has concentrated its resources and expertise at all levels on this package.

The working week typically followed by these students is represented by Figure 2 below. The learning, teaching and assessment strategy adopted is geared towards the development of a high level of independence in the student as a prelude to lifelong learning. This philosophy is reflected in the organisation of the taught work with students being encouraged to organise self-help groups and request surgeries from staff to support them. In addition, those students who find that they lack the necessary background for successful study are encouraged to use appropriate *mathwise* modules to support their studies. Much of this work takes place outside of the normal timetabled hours which, in the case of Sunderland students, occupy some 60 hours of a 150 hour 20 credit module.

The variety of assignments required of these students has already been mentioned and the following problem which has been used with first year Combined Programme classes is closely based on problems which may be found in the text "Discovering Calculus with Maple" by Harris and Lopez (2E 1995).

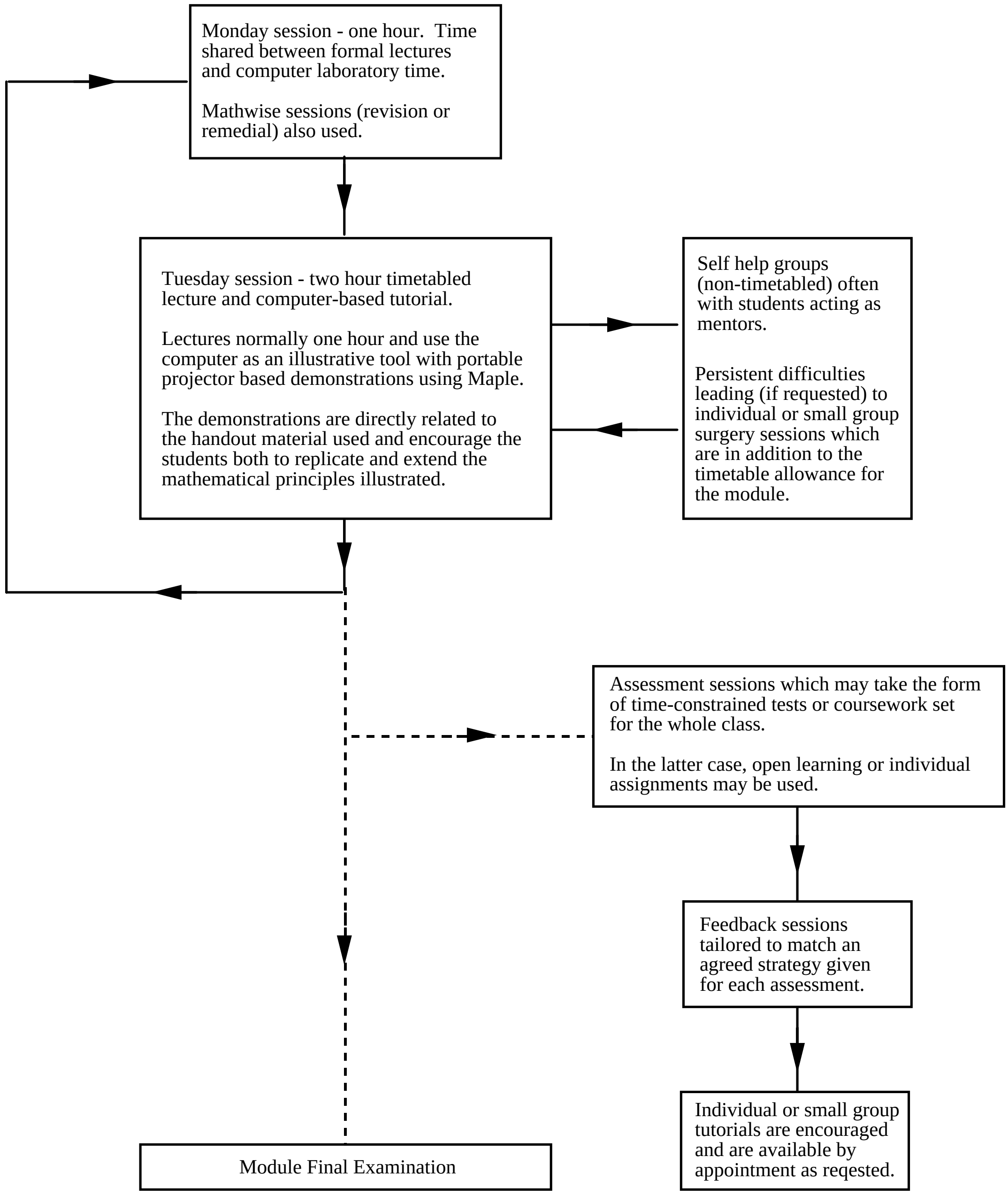

Figure 2 - Typical Weekly Organisation

3.3.1 Calculus Problem

Introduction

In this assignment you are given functions of x and you are asked to calculate the value(s) of the variable x for which turning values of the function occur. You are also asked to demonstrate the occurrence of the turning values both graphically and numerically and to explain why the results obtained by the different methods match. Remember that turning points may be classed as: maxima, minima or inflexions, you will meet all of these types in this assignment.

Part 1

Consider the polynomial

$$f(x) = 12x^5 - 120x^4 + 400x^3 - 480x^2 + 5$$

a) Calculate the values of the variable *x* which determine the turning values of the function.

b) For each distinct value of *x*, say $x = a$, define the function

$$g(h) = f(a) - f(a+h)$$

and compute the values of $g(h)$ for several small values of *h* both positive and negative.

c) Use the computed values of $g(h)$ to classify each $x = a$ as corresponding to a local maxima, a local minima or an inflexion.

d) Plot the function $f(x)$ over a suitable range of values of *x* and explain how the plot and the results you obtained in parts b) and c) correspond.

e) Use Maple (or hand methods) to express the derivative of the function $f(x)$ in a fully factored form. You will find that the expression you obtain will contain a squared term. Explain why it is that the turning value associated with this term cannot be a local maximum or minimum.

Part 2

Consider the function

$$g(x) = e^x + \frac{1}{5}\cos(20x)$$

a) By plotting the function $g(x)$ for the range of values from $0.95 \cdots 1.00$ *estimate* the value $x = a$ such that $0.95 < a < 1.00$ where $x = a$ corresponding to a turning value. Work to two decimal places if possible.

b) Find, correct to 4 decimal places, the value of the variable $x = a$ such that $0.95 < a < 1.00$ where $x = a$ corresponding to a turning value and explain why it might be difficult for you to do this without the aid of a computer.

c) Define the function

$$i(h) = g(a) - g(a+h)$$

and compute the values of $i(h)$ for several small values of h both positive and negative.

d) Use the results you have obtained in part c) to predict whether the function $g(x)$ has a local maximum or a local minimum at $x = a$.

e) Using both your estimated and calculated values of $x = a$, calculate the approximate percentage error in the value of the function $f(x)$ at its turning value in the interval $0.95 < x < 1.00$.

While the results obtained from this assignment have, as always, varied from excellent to poor, the assignment has succeeded in stimulating considerable discussion among student self-help groups as to the behaviour of functions close to any turning values which may exist. From this point of view the example has proved instructive. The author has used group based assessment methods with other (service) groups of students, often with variable success. The scenario outlined below is an example of a group assignment which did bring problems in its wake.

4 Group Based Assessment

The author believes that an investigative approach to the teaching of mathematics should be supported by a similar approach to its assessment. Instead of a formal examination, group work in which staff, self and peer assessments are combined to give the final grades was used. Not all of this work was entirely successful. Some of the work set, although successful in some ways, did draw very negative comments from the authors peers, particularly those from outside the University. An example is given below.

Introduction

The computer algebra package DERIVE *is available for use for any part of this assignment and you will be expected to use the facilities of the library and organised your work as a group appropriately. The group as a whole is responsible for the report which will be marked by the member of staff responsible for the delivery of the module. Your final mark will be determined by a combination of the following assessments.*

- *a) Self Assessment;*
- *b) Peer Assessment;*
- *c) Group report mark.*

Assess the contribution made to the assignment by both yourself and your peers using a scale from 0 - 10. Take 0 to mean that no effort at all was put into the assignment and 10 to mean that the contribution of the individual concerned could not have been higher. The formula used to determine the final grade for each student will be as follows:-

$$\begin{aligned}\text{Final mark} &= \text{Weighting} \times (\text{Group\%}) \\ &= (\text{Average of } (\text{Self+Peer Assessments})) \times (\text{Group\%})\end{aligned}$$

Each report must be accompanied by a sealed envelope from each student containing the self and peer assessment grades according to the above rules. Note that the highest and lowest grades given will be ignored in the final grading calculation.

Part 1

Author the function $\frac{1}{1-x}$ and by repeatedly differentiating it show that the n^{th} derivative of the function is given by the expression

$$\frac{d^n}{dx^n}\left(\frac{1}{1-x}\right)=\frac{n!}{(1-x)^{n+1}}$$

where $n!$ is called "factorial *n*" and is calculated from the formula $n!=1.2.3.4.\cdots n$, so that, for example

$$2!=1.2=2$$
$$3!=1.2.3=6$$
$$4!=1.2.3.4=24$$

and so on.

It is important to note that $0!$ is defined as 1. You may need this information later in the assignment.

Part 2

Find an expression for the n^{th} derivative of the function

$$\frac{x}{(1-x)(1-2x)}$$

Hint:- Expand the function $\frac{x}{(1-x)(1-2x)}$ and treat each part separately. Develop and write down an expression for the n^{th} derivative of each term.

Part 3

Use a similar method to those you have used above to find an expression for the n^{th} derivative of the function

$$\frac{1}{1-x^2}$$

Part 4

By finding the appropriate derivatives and plotting the appropriate graphs, show that the function

$$\frac{d^n}{dx^n}\left(\frac{1}{1-x^2}\right)$$

has a turning value at the point $(0, n!)$ provided that *n* is an even number and identify its type. Does the same result apply if *n* is an odd number? Justify your answer.

4.1 Results

This assignment was set only twice, but the author believes, after discussion with the participating groups of students, that the perception of investigative group assignments as a vehicle for offering more than mathematics alone did have a beneficial effect on the overall learning experience of the student.

Student reaction to their integral involvement in the assessment process and the extra demands made on them because of this did result in their re-thinking the purposes of assessment and its responsibilities. Students had, at first hand, had to cope with the responsibilities of:

- trying to differentiate on an honest basis between the relative merits of the members of their group;
- trying to give an honest assessment of their own worth in relation to the overall group effort and its products;
- deciding how best to deal with those members of the group whose effort was perceived as lacking in some way;
- acting collectively and dealing with any members of their group who simply did not contribute to the completion of the set task. It was made clear that such group action may result in an individual student failing.

The realisation that group assessment can offer more than a traditional examination did encourage the students to accept roles in which transferable skills such as:

- IT-based skills;
- information retrieval skills;
- problem-solving skills;
- communication skills;
- interactional skills;
- project planning and control skills;

are acquired and to develop the sense of intellectual honesty essential to their participation in the assessment process.

Some results obtained from the assignment outlined above are shown below.

GROUP 1

NAME	Self and Peer Assessment					Weighting	Group Assessment	Final Mark
	S	**P**	**P**	**P**	**P**			
Student 1	7	?	8	7	8	75	92	69
Student 2	7	7	8	?	8	75	92	69
Student 3	?	6	5	6	6	58	92	53
Student 4	7	6	7	6	?	65	92	60
Student 5	8	8	?	8	8	80	92	74

GROUP 2

NAME	Self and Peer Assessment						Weighting	Group Assessment	Final Mark
	S	**P**	**P**	**P**	**P**	**P**			
Student 1	9	6	8	8	-	-	78	80	62
Student 2	8	9	8	8	-	-	83	80	66
Student 3	-	-	-	-	-	-	-	-	0
Student 4	-	-	-	-	-	-	-	-	0
Student 5	7	9	8	8	-	-	80	80	64
Student 6	8	9	7	8	-	-	80	80	64

GROUP 3

NAME	Self and Peer Assessment						Weighting	Group Assessment	Final Mark
	S	**P**	**P**	**P**	**P**	**P**			
Student 1	8	7	9	8	7	**5**	78	70	55
Student 2	8	9	9	8	8	9	85	70	60
Student 3	6	9.5	9	9	9	10	88	70	62
Student 4	9	9	9	9	8	**5**	88	70	62
Student 5	?	9	9	8	9	**5**	88	70	62
Student 6	?	8.5	7	9	7	**5**	79	70	55

Notes

1) With the exceptions outlined in 3 and 4 below, all of the grades allotted by the students were used to calculate final grades;
2) group 1, student 3 submitted no self assessments while others omitted some peer assessments;
3) group 2, students 3 and 4 did not take part in the assessment and were withdrawn by the rest of the group. This resulted in fail grade being given to the students concerned;
4) group 2, student 1 was the weakest student in the group yet gave herself a high self assessment mark. In contrast, student 5 was the strongest but gave himself a low self assessment mark;
5) group 3, students 5 and 6 did not submit self assessments. The marks of 5 were deemed (by the author) to be in sufficient disagreement with the remaining submissions and were neglected in the final grade calculations. It is difficult to know whether or not to include all

assessments given and difficult to establish the precise criteria by which some assessments may be neglected;

6) in both semesters during which the module was delivered, student attendance levels and enthusiasm remained high. This extended to the completion of the assignment. Viewed from this perspective the assignment must be viewed as a successful vehicle for stimulating mathematical discussions and effort with students whose mathematical background is a poor GCSE grade;
7) due to the lack of external peer approval for this assignment, it was dropped in favour of a time constrained test after the second occasion on which it was used.

5 Some Lessons Learned

Firstly, in terms of the coursework set for non-specialist students, it can be difficult to balance desirable mathematical outcomes with the demands from the host department for the development of transferable skills. Other than quoting the obvious argument that mathematics itself is a valuable, useful and desirable transferable skill, one may consider the overall benefits gained by students whose exposure to mathematics during their time in Higher Education is limited to a relatively few hours addressing a first year service course. The author has found himself faced with issues whose resolution in favour of the long term benefit to the student is far from obvious, for example:

- should the lecturer insist on developing (say) the calculus from first principles, including a discussion of continuity and limits, knowing that many students in the class may not, at least over the available time-scale, understand the aims and objectives of such a discussion;
- should the lecturer insist that students attempt to attain even a basic familiarity with algebraic manipulation knowing that many of them have failed to assimilate even the rudiments of the skill during the course of their education at school level and that the passage of even a short time after the completion of their university mathematics module(s) will negate any gains made;
- should the lecturer attempt to concentrate on applications of the subject intended to appeal to students knowing that many of them will not have, nor will they be able to attain, the necessary background to facilitate understanding;
- should the lecturer sacrifice some mathematics in order for supposed gains in other more general areas such as Information and Communication Technology (ICT) supported methods in learning and teaching, or experience of judging the effort and quality of the work of their peers in group-work situations;
- should the lecturer admit that school mathematics has failed many students and embark on a course of action which deliberately avoids the teaching methods used previously;
- given that many students are now positively anti-mathematics should further mathematical content and precision be sacrificed for mathematical activities which students claim as enjoyable?

It is perhaps worth remembering that the group-based activity described in Section 4 above was, from a student perspective, a positive and enjoyable experience in spite of the fact that (external) colleagues had severe doubts as to its worth, some being of the opinion that “they will learn nothing about mathematics by pursuing such exercises.”

In essence the author is asking whether it is better to risk even further mathematical alienation or to attempt to ensure that students, perhaps through the use of technology, experience a course in mathematics which they consider to be enjoyable even at the expense of some mathematical desirability? In terms of service teaching, the author suggests that the latter course of action should at least be considered.

Secondly, over the past decade, several different methods of conducting formal examinations have been tried and have met with varying levels of success. Two common threads have been evident throughout every method tried, these are:

- a desktop computer equipped with either DERIVE or Maple has been available for student use throughout the examination;
- the total time allocated to any given examination has been extended to allow for activities such as printing and the transfer on on-screen information to answer booklets.

Examinations designed for use in a situation where students had immediate access to an algebra package were first used in the academic year 1992-93. The content of the examinations was much less reliant on the restrictions often governing traditional examinations in that (for example) manipulation, graph plotting and exploration leading to generalised results became much easier to deal with and it became possible to introduce scenarios whose purpose was to make higher mathematical demands on the students than had previously been the case. The organisation of these examinations (which coincided with the University adopting both semesters and a modular credit scheme simultaneously) is outlined below:

- students were allowed the “standard” three hour thinking and doing time;
- a specially designed answer booklet was used whose purpose was to ensure that students submitted their answers in writing, adequate space being provided for the inclusion of computer input and output and sketches of on-screen graphs should the student deem this necessary;
- students were encouraged to make a decision as to how to attempt questions and/or parts of questions in the sense that they were able to decide whether or not the computer was an appropriate tool for the task in hand;
- students had to decide how to construct their answers using a combination of traditional and computer-based methods.

It quickly became apparent that:

- if students were to be given the “standard” time allocation for an examination, considerable additional time had to be allowed for printing particularly as students were using networked printers which were relatively slow;
- time allowances had also to be made for technical problems such as machine crashes and failures;
- the specially prepared answer booklets were not necessary and offered no advantages over the booklets provided by the University.

As a result of this experience, staff considered the implications of allowing students to submit their answers entirely electronically on a floppy disk. As a result of experience gained in trying to implement this method of submission with coursework and short (one hour) time constrained tests, the method was rejected for the following reasons:

- disks may become corrupted or virus-ridden;
- students may fail to save all of their work while acting under the stress of examinations;
- at second and third year level, paper-based answers were preferred by external examiners for moderation purposes.

More recently, second and third year students have sat computer-based examinations using Maple running on SUN machines. At Sunderland this has the following advantages over the PC's provided for student use:

- the SUN machines used do not have a floppy disk facility and so the possibility that the machines become infected by a virus transmitted from a disk owned by the student does not pose a threat;
- the UNIX machines used are far more stable than the Windows 95/98 PC's used;
- students are given their own disk space allowance which is usually (but not always) adequate for their purposes;
- the many students from other schools within the University who generally take modules which essentially involve computer basics do not use SUN machines thus making fewer demands on this system.

Generally, this has worked well and has the advantage that the work done by the student is saved for a short time after the examination on the server associated with the system used for the examination. However, it should be noted that the system is not foolproof as the following example shows.

During the 1999-00 semester one examination for second level Combined Programme students, no problems were met until the end of the three hour period allowed and students stated to print those sections of their work to be handed in for marking. It should be noted that this varies widely according to student preference with Sunderland students generally falling into one of the following categories:

- those who use the computer to the exclusion of any other method and print off their complete answers. These students usually include notes and explanations as part of their Maple files and so do not use answer books in the conventional sense;
- those who write answers in the answer books provided but supplement these answers with complex diagrams (three dimensional graphs and phase portraits for example) produced by Maple;
- those who use the answer books provided to address proofs, hand-written comments and the results of exploration scenarios to the examiners but print all other work.

At the end of the examination, two students falling into the first category attempted to print off their work but could not accomplish this. The problems were eventually traced by technical staff and were explained as follows:

- one student had almost reached the limit of his allowed disk space and while his file was saved, printing it was not possible until the technicians had increase the limit temporarily. This involved closing the student account but the situation was dealt with. *It should be realised that the system gave no warning of impending disk space problems;*
- the second student completed his examination but again could not print. This problem was eventually traced to the student inadvertently exceeding his account with the result that the Save command, although appearing to function normally, was not in fact saving the file! *Again, the system failed to warn the student of this problem*! This student had to face the inconvenience of attempting to recreate his answers after permission to do this had been obtained from the appropriate external examiner.

Since the occurrence of the problems alluded to above, the author has asked his technical support staff to ensure that special, temporary, examination accounts are set up with generous capacities to ensure that the problem does not repeat itself in the future.

6 Conclusions

The use of computer-based examination regimes does allow the use of commonplace mathematical tools in all aspects of the learning, teaching and assessment situations in which modern students are placed. On the basis of his current experience, the author suggests that colleagues intending to allow students full access to computers for examination purposes consider the following guidelines:

- ensure that either, the machines do not allow student access via a floppy disk or that they are as fully protected as possible by anti-virus software and configured to prevent user access to the operating system;
- ensure that the machines are professionally maintained by technicians who are responsible for verifying the condition of the machines prior to any examination;
- ensure that the technical staff responsible for the maintenance of the machines configure special examination accounts with a generous, if temporary, capacity;
- ensure that any accounts set up for examination purposes are protected to allow only staff access until at least the end of the marking process and preferably longer to allow staff to access information in the event of a student appealing against any grade given;
- ensure, if possible, that fast laser printing facilities are available during the course of the examination and that any special instructions regarding printing are clearly and permanently displayed to the students. This applies particularly if students have to use a computing facility with which they are unfamiliar;
- once a set of machines has been verified, ensure that there is no access to them before the examination is due to commence;
- ensure that, if possible, the machines either save work regularly and automatically, say every ten minutes, or that the student is prompted, over a similar time interval, to manually save any work done;
- ensure that, within the examination environment, there is an over-supply of machines so that any machine failures experienced during the examination period can be dealt with immediately. A figure of 10%-15% is suggested;
- ensure that any invigilators involved in the examination process are reasonably familiar with both the packages being used and the basics of system running them;
- ensure that immediate technical assistance is available should this prove necessary.

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