Fourth International Derive TI-89/92 Conference
Liverpool John Moores University, July 12 – 15, 20000

Estimating Time Since Death[1]

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Introduction

The scene is a familiar one to any person who has watched murder mystery movies. The coroner is called to the witness stand or is questioned by the detectives investigating a murder case. "At what time did the victim die?" The coroner replies, "Between the hours of 10:45 and 11:00 PM on the night of ....." The time of death is indeed an important piece of evidence used in the investigation of a homicide. Much was made of the time of death in the famous O.J. Simpson Trial. Yet, the noted forensic expert Bernard Knight [1; page 2] says that any estimate of the time of death based solely on physical evidence and which does not have a variance of at least one half hour in either direction (an interval of at least one hour) should be viewed as suspect and not trusted. In the O.J. Simpson Trial the estimates of the time of death were based on witness reports and not physical evidence.

It is also commonly agreed that the classical temperature based methods are among those prone to the greatest error. These methods are based on Newton's Law of Cooling. Many times first year calculus and also differential equations instructors will assign a problem that has the setting of a murder victim being found and the initial temperature for the body is given along with a coefficient for the cooling of a body. Students are led to believe that they are dealing with a real world application of Newton's Law. In fact, their estimate derived in this way may be incorrect by several hours.

Knight [1] devotes the first two chapters after a general introduction to temperature based methods and gives a history of their use and certain developments in the evolution of the use of this method. At the end of our presentation, we will give yet another modification of the method that may help to make the application of the method more realistic. However, our main purpose in this presentation is to give instructors who want to discuss the application of Newton's Method to estimating time since death some background on how a death investigation is conducted and the duties of the coroner with respect to a death investigation.

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[1] This paper was first presented at the International Conference of Teachers Teaching with Technology held in Dallas Texas, March 16 – 19, 2000 and will appear in the Proceedings of that conference

We begin with a brief discussion of the history of the coroner's office and how the office is established in the state of Pennsylvania. We then proceed to the duties of the coroner, conduct of the coroner on the scene of an investigation, and the means used to estimate time of death. We conclude with the construction, testing, and application of a mathematical model for making a temperature based estimate.

History of the Coroner's Office

While some mention of the office of Coroner may be found in earlier records, say during the reign of Alfred the Great, the office of Coroner as we know it today was instituted during the reign of Richard the Lionheart. Contrary to popular belief reinforced by the Robin Hood movies, Richard was not a good and kindly king. He was interested only in making war and spent little time in England. In his view, England was only a source of revenue. He and his wards instituted heavy taxes on the English and had Sheriffs in each "shire" to collect taxes and keep his coffers filled. The system was riddled with corruption – one true aspect of the Robin Hood films. Sheriffs were skimming much of the revenue for themselves, and the revenue was not flowing into the king's coffers.

It is also true that Richard was held in Germany for a huge ransom. His faithful minister Hugo Walter came to the Continent to arrange for a ransom, an amount of 150,000 marks which is equivalent to several million dollars in today's terms. Walter who eventually became the Chief Justiciar and Archbishop of Canterbury was the de facto ruler of England (Richard only spent a total of four months there). He replaced many of the sheriffs, but realised that it was only a matter of time before the new sheriffs may emulate their predecessors. To keep money flowing into the King's Coffers he revived the old office of the Coroner.

The legal foundation of the office is recorded by Knight [2] in an extensive history of the office given on the Web. According to Knight:

> *"Now the actual formation of the office of Coroner is based on an extremely skimpy base. The edict that formally established the Coroners was Article 20 of the "Articles of Eyre" in September 1194. The "General Eyre" was the periodic visitation of the King's itinerant Judges, who travelled slowly around the country dispensing what passed for justice in those days. It was the forerunner of the "Assizes", derived from the Norman-French for "sittings" which, in turn, of course, gave way to the present Crown Courts in more recent times. The Eyre of September 1194 was held in the County of Kent, and Article 20 baldly stated that*:
>
> > "IN EVERY COUNTY OF THE KING'S REALM SHALL BE ELECTED THREE KNIGHTS AND ONE CLERK, TO KEEP THE PLEAS OF THE CROWN"
>
> *And that is the only statutory basis for the Coroner. Each county had three Coroners and a poor man who had to walk behind their horses, carrying the "Coroners' Rolls" and pen and ink: a Medieval Coroner's Officer, you might call him; though even this minor office was abolished in later years to provide for another Coroner. Now the above words are the only official authority for the long-lived system and, looking at the remit of the new Coroners,*

> *they were ordered only to "keep the pleas of the Crown". This meant recording the pleas on parchments known as the "Coroners' Rolls," many of which survive today in the Public Record Office.*
> *"Keeping the pleas" was quite different to "holding the pleas" which meant actually trying the cases and passing sentence. This could only be done for lesser offences at the County Courts by the Sheriffs, otherwise the cases had to be committed to the next General Eyre when it trundled along in the fullness of time - which might be years ahead. In fact both Coroners and Sheriffs did, in the early years, hold pleas of the Crown, acting as Judges in an ultra vires fashion. Consequently, one of the demands of Magna Carta, some twenty years later, expressly forbade this practice. Chapter 24 of Magna Carta states: "No sheriff, constable, coroner or bailiff shall hold pleas of our Crown".*
>
> *The keeping of the pleas of the Crown was the source of the title, the original Latin was "custos placitorum coronas" from which the word "coroner" is derived. He was referred to for hundreds of years as "the Crowner" - as in Shakespeare's Hamlet, where derisively it is said "But is this law? Ay, marry, is't crowner's quest law!"*

So the office was established and named. The requirement of being a knight gave way to being among the most “meet and lawful men of the county.” However, the main duty was to see that the King received his just income from the population, his subjects. One of the large sources of income was any death that occurred in the county, and death investigation, the only remaining duty of the Coroner, whether suspicious, accidental, or natural became an important duty of the Coroner. The Coroner was to establish the circumstances surrounding the death. The General Eyre which reviewed the records perhaps years later then could fine the population for any wrong procedures. Thus, the keeping of accurate records was essential to the Coroner’s duties as well as the Crown’s income.

The Office of the Coroner became firmly established in the English Civil and Jurisprudence system and has evolved to the Office as we know it today. It was brought to the Colony of Pennsylvania, or “Penn’s Woods” with the early settlers. In the late 1600’s the procedure for filling the office was that the freemen of each county annually nominated two individuals and the governor appointed one of them. The Constitution of 1776 continued the annual appointment, but the Constitution of 1790 provided for a three year appointment. In 1838 the office became elective. In 1909 the term was extended to four years.

The duties of the Coroner have remained as defined in the 1680’s and recorded in the Pennsylvania State Archives records on County Offices [3]. The Coroner is responsible for investigating sudden, suspicious, or violent deaths in the county. The duties of the Coroner are to issue certificates indicating probable cause of death, holding inquests if necessary, and overseeing the county morgue. Pursuant to these duties the Coroner is authorised to have autopsies performed, hold inquests, summon jurors and subpoena witnesses. The Coroner’s records are usually to include dockets and investigatory files generally consisting of autopsy and toxicology reports, correspondence, inquisition sheets, lists of personal effects, notes, photographs, and summary reports.

The requirements for holding the office of Coroner are that the person be 21 years of age and a United States Citizen. The Coroner must be elected by the citizens of the county and must be a resident of the county for one year prior to election. Recently a requirement has been made that the Coroner must take a course of one week in length at the State Police Academy or have equivalent training. Prior to assuming office the Coroner must pass an exam administered by the State Attorney General's Office. Once in office the Coroner may appoint Deputies that have the same power as the Coroner. The Deputies are approved by the County Commissioners.

The Certificate of Death issued by the State and signed by the Coroner requires that a date of death be established. It also requires that the Coroner establish a manner and cause or the death. It may be important in establishing these facts that a time of death be established. This time may also be useful in any ensuing investigation by law enforcement officials.

The Coroner on the Scene of a Death Investigation

Since the duties of the office of Coroner require the establishment of the manner and cause of death, (Is this an accidental death, natural death, suicide, or homicide?) and these determinations can have profound effects, it is essential that the Coroner have full charge of the scene of a death. This includes having authority over and the use of emergency services' personnel, police, and other people in attendance at the scene. The Coroner needs to develop a clear picture of the scene and all facts related to the death of an individual. She needs to know in what way the scene has been altered by the entrance of investigating officers and emergency personnel. Pictures need to be taken. Onlookers and witnesses need to be questioned and facts concerning the case need to be determined.

The Coroner needs to take charge of the possessions of the deceased until they can be dispersed to a responsible family member or an appropriate individual. In general, it is the Coroner's duty to uphold the rights of the deceased or to put it in a more popular phrase, "to speak for the deceased." A large part of this responsibility is to guarantee that all facts related to the death of the individual are known. Thus, the Coroner must complete a thorough and rigorous investigation of the circumstances surrounding the death. This may include the best possible estimation of the time of the death.

In addition to protecting the rights of the deceased, the Coroner needs to complete public documents and meet the public's "right to know." This means that the Coroner must respond to reasonable requests by the public press and be prepared to present evidence at any legal inquiries. The Coroner generally needs to co-operate with investigating authorities and not obstruct them in their investigations. At the same time this co-operation needs to be balanced with the need to protect the rights of the deceased and the family of the deceased. Sometimes the protection of these rights may be set by law, such as in the case of HIV infection or other diseases where the rights of privacy of the deceased are involved.

Means of Estimating Time of Death

There are several means for estimating the time of death. Of course if the event is witnessed, the time of death can be established with accuracy. This is true in the hospital setting, an attended death at

home, or many times in the case of many motor vehicle accidents and other accidents such as industrial or work related accidents. In other cases witness encounters with the deceased prior to and after death can be used to set bounds on the time of death. One, somewhat humorous, situation occurred when the second author made a rough heat loss based estimate for the first author, and she replied, "That can't be, the person was seen alive by a friend two hours after that time." Such is the problem with using mathematical equations to establish the time of death.

One, reasonably accurate estimate in the case of a fairly recent death is to measure the potassium level in the vitreous fluid of the eye. It this level is denoted as K_v, then the formula:

$$7.14 \times K_v - 39.1$$

gives a reasonable estimate during the first nine and one half hours after death. Of course, obtaining the vitreous fluid is a medical procedure and is generally done at a place other than the scene of the incident.

There are several physical signs that can assist the Coroner in establishing bounds on the time that has elapsed since the time of death. The following is a list of a few of these signs. For the most part they are used to check consistency of other methods.

Sign	Lower Bound (Hrs.)	Upper Bound (Hrs.)
Tendon reflexes present	0	2
Dried Blood (wounds) estimate from drying at periphery inward	½	4
Livor mortis-mild (pink coloration at lower portions of the body)	½	4
Livor mortis-livid	8	12
Rigor mortis-onset (subtract one hour for each 10^o F above or below 70^o)		
Jaw muscles	2	6
Neck and fingers	3	7
Wrists	4	8
Elbows	5	9
Shoulders and knees	6	10
Hips	7	11
Abdomen (complete rigor mortis)	8	12
Rigor mortis disappearance (subtract one hour for each 10^o F above 70^o and add one hour for each 10^o below 70^o		
Jaw muscles	26	30
Neck and fingers	27	31
Wrists	28	32
Elbows	29	33

Shoulders and knees	30	34
Hips	31	35
Abdomen (complete rigor mortis)	31	36

There are other physical signs that can also be used, but the above table gives some of the most used signs. As can be seen these signs do not give exact times and really, as previously mentioned, only establish bounds and give credibility to other measurements.

This brings us to the issue of using a temperature-based method and the use of Newton's Law of Cooling. There are several sites that have been used over the years. Some used a measurement of skin temperature because of the non-invasive nature of this technique. However, the measurements are affected by changes in ambient conditions, the size and age of the patient, the amount of clothing worn by the patient, etc. In general these measurements have a tendency to be inaccurate. Another only mildly invasive site is the rectum of the victim. Knight [1] points out that in many cases there is actually a rise in the measured body temperature after death using this site. The reason for this is the fact that the organs of the body discharge heat immediately after death. This is partially due to the inhomogeneous nature of the body and the fact that heat escaping from the various organs will take a most direct route. Knight advocates the use of trachea measurements, but this is not easily accomplished in the field.

A sight that is a homogeneous organ, whose access is only mildly invasive, and that can be accessed with relative easy in the field is the liver of the deceased. It is accessible via a small incision made in the abdomen and a stainless steel heat probe similar to that used in the new CBL2 kit. The liver is a protected organ and tends to loose heat in a uniform way. It is not as subject to the other confounding influences as the other methods tend to be. The standard procedure is to monitor the temperature at intervals of 15 minutes to establish the cooling characteristics of the organ for the individual person.

Having discussed the general nature of a death investigation and the obtaining of information that can be used to estimate the time since death, we will proceed to a modelling technique which can be applied to the data gathered by the coroner.

A Mathematical Model Based on Heat Loss

Many of us have at some time or other run the following temperature experiment in some of our classes. We have a cup of hot liquid. Many instructors use their cup of coffee to insert a bit of a personal touch into the class. Then as we are developing the solution of the differential equation resulting from Newton's Law of Cooling, we have a temperature probe in the cup of liquid measuring the temperature of the liquid at precisely spaced intervals. At the conclusion of the exegesis and the experiment, the theoretical and experimental results are compared. Usually the experiment is run for times anywhere from one half to one hour. The comparison of the theoretical and experimental results is generally very impressive.

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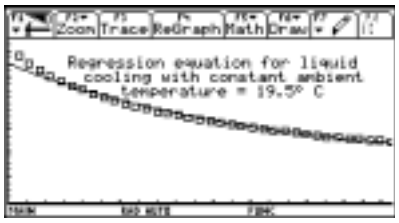
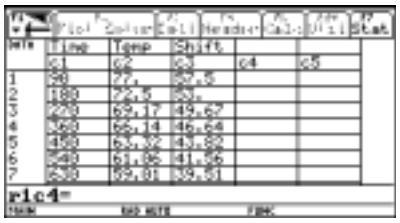




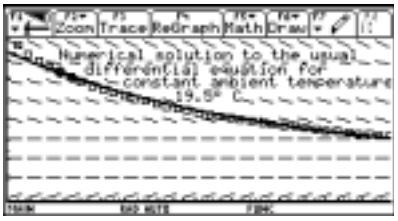
Below are the results of an experiment that was run in a kitchen with time measurements being made every 90 seconds. On the left is the data as it was gathered and on the right is a plot of the data together with the graph of an exponential function obtained by doing a least squares fit of the observed temperatures less the ambient temperature versus time.

As is noted in the figure on the left the ambient temperature was 19.5^{o} C. The regression equation for the given data is T(t) = 53.25 * $.995298^{t}$ + 19.5 Taking the natural logarithm of .995298 and considering Newton's Law of Cooling with a constant temperature

$$\frac{dT}{dt} = -.00047(T - 19.5)$$

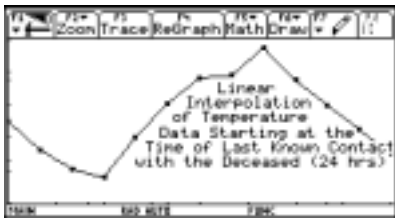
$$\frac{dT}{dt} = k(T - T_a)$$

We have the differential equation:
Using the Differential Equations mode of the TI - 92 plus we plot the slope field and the data points. We graph a numerical solution starting at one of the intermediate data points and observe that the agreement with the experimental data is uncanny.



This is one of the reasons we use this experiment in our classes. It gives a good solid application of the solution of the differential equation to the real world phenomenon that the class observed.

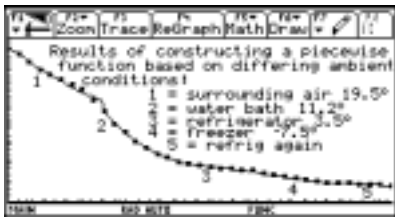
But, what about the application of this differential equation to different situations? The equation given above assumes a constant ambient temperature. What if the ambient temperature varies as the body is cooling? In this case we have a non-linear differential equation and an analytic solution may not be possible. For example, suppose the coroner is called to the scene of a body found outside and the last time the person was seen was several hours previous to the discovery of their body? For example, suppose an individual left a group of friends at midnight and was declared missing some hours later. The person's body was found in a park the following evening and the coroner arrives at 8PM. Temperature data exists for from midnight until the time of discovery. Here is a graph of the ambient temperature data. Note that measurements were only taken every two hours.



The range on this graph is from 25° F to 45° F. We use the Fahrenheit scale here because of the fact that we will eventually be asking when was the body temperature 98.6° F? Obviously, we can not assume that the body cooled under the condition of a constant ambient temperature. We will use the differential equation for Newton's Law of Cooling in this case, but in a highly non-linear form. The numeric differential equations solver of the TI - 92 plus will, however, be able to handle the equation.

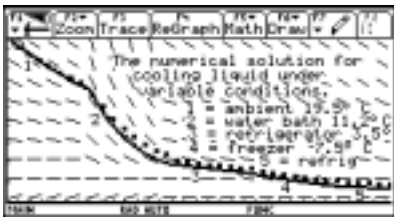
Prior to discussing our hypothetical case, we will first run one more experiment. It will be a stepping stone between our original experiment and the coroner's problem. For this experiment, once again a cup of liquid is used, but the cooling conditions are varied. Six cooling strategies are employed during the 54 minute cooling period. First the liquid is cooled as before in the ambient surroundings of 19.5° C. Then the vessel is put into a water bath at 11.2° C. Next it is placed in the refrigerator at 3.5° C, the freezer at -7.5° C, and finally, returned to the refrigerator. During each of the ensuing time periods the ambient temperature remains constant. Thus, we can use a standard exponential regression to describe the temperature of the liquid during each of these periods. However, for the differential equation model, we will use a piecewise-defined function for the ambient temperature given over the entire period.

We first graph the data as it was collected and then show the differential equation model.



We need to say a word about region 2 of this graph. This is during the time when the vessel of liquid was placed in a water bath. The bath was crudely constructed with a baking dish in a sink with the cold water running very slowly. Some of the water very likely splashed into the vessel of liquid changing the cooling characteristics of the liquid. This accounts for region 2 having a more pronounced slope than the other parts of the graph. We needed to account for the different cooling characteristics in the description of the differential equation for the cooling. Here is our non-linear model for the process.

$$y' = \text{chi}(t,0,694)*-.000775(y - 19{,}5) + \text{chi}(t,695,1349)*-.002583(y - 11.20) + \text{chi}(t,1350,1979)*-.0005(y - 3.5) + \text{chi}(t,1980,2789)*-.00046(y + 7.5) + \text{chi}(t,2790,3240)*-.0006(y - 3.5)$$



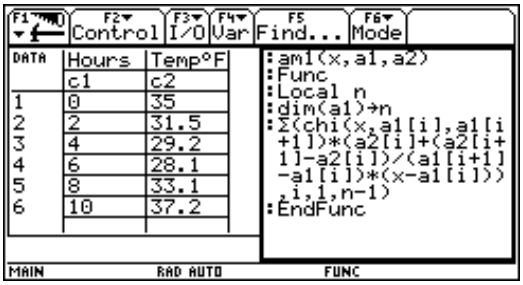
Note how the cooling characteristics of the liquid changed during the water bath and after the water bath. Previous to the water bath k had a value of –.000775. After the water bath it was closer to –.0005. This points to sloppy experimental procedure, but one which serendipitously gives us a more interesting function to consider. Here is the graph of the slope field and a numerical solution starting with our initial temperature reading

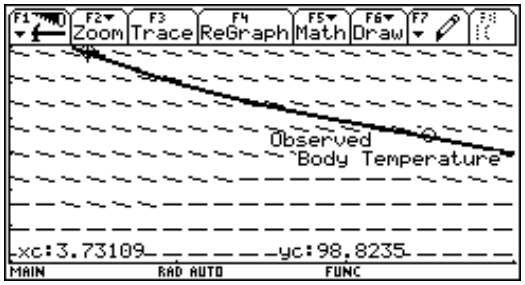
The agreement with the data is not exact, but it is quite close and justifies our confidence in using this method for solving the coroner's problem.

Let's return to the coroner's case. The victim was last seen alive at 12 Midnight and the body is found much later. The coroner arrives on the scene at 8PM. She takes a liver temperature of 70^{o} F and takes further temperatures at 15-min intervals for an hour to establish the cooling characteristics of the liver. Her data will justify a value of $k = -.03$. This is basically all that can be done on the scene relative to determining a temperature-based estimate of the time since death. Upon returning to her office she is able to get temperature data from her local weather station. She hands all of her data to a mathematician for analysis.

The data is placed in a table and a linear interpolation formula is developed to describe the temperature function over the 24-hour period since the victim was last reported to be alive. (Note we only need it over a 20-hour period, but we use all of the data). This function is shown in the graph above that follows the first mention of the problem.

Using this function we set up the differential equation: $y' = -.03(y - am1(t))$ where $am1(t)$ is the function interpolating the temperature data. The data and the graph of the slope field which include a graph of the numerical solution of the coroner's data shown below. We can also see from the trace the approximate time when death occurred.





From this result we can assume that the death occurred somewhere around 4AM. Once again, we stress that this is only a rough estimate. Other evidence will need to be considered to support our finding. If the victim came to a violent end, it gives investigators an interval, say from 3AM to 5AM, to begin questioning the whereabouts of suspects and possible material witnesses.

References

1. Knight, Bernard (editor), The Estimation of the Time Since Death in the Early Post-Mortem Period; Edward Arnold, Great Britain, 1995,

2. Knight Bernard, "History of the Medieval Coroner System" as found on the World Wide Web at the URL http://www.britannia.com/history/articles/coroner1.html

3. Pennsylvania State Archives – County Offices, as found on the World Wide Web at the URL http://www.state.pa.us/PA_Exec/Historical_Museum/DAM/rg/offices.htm