Creativity with the TI-89

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Change to a new style

Mathematical education has been changed to a new style by the new technology, computers and the graphic calculators in mathematical classroom. In this new mathematical education, what is important to students? In Japan we think that it is important to study the method of the calculations in mathematical classes. Furthermore, we used to think the ability to do mathematics is equal to the doing the calculation. In the new system, the ability to do mathematics is creativity in mathematical thinking. The calculation is done by technology. The technology is the tool for students in mathematical class. They are thinking with the tool. The tool is very good for thinking mathematically. It is very useful for thinking about and doing mathematics. In this time it is important to be creative with mathematics. In the future, the purpose of mathematical education will be to develop creativity in the mathematics. Creativity with mathematics will be the goal for mathematics class lessons. So the method of mathematical education will be changed to the new style by the new technology, the computer and the graphic calculator. At present we can not change the style of the education with new technology in Japanese school. I want to change the educational style in collegiate mathematics. I am using the graphic calculator, the TI-89, in my mathematical lessons for every student. I want to say that my students think and make good theorems during this class. This direction of mathematical education is doing mathematics and creativity is the main Idea. At present, making theorems is not a new experience for the students. It is not a problem, the students are repeating the mathematical theorems which were created by old mathematicians. I think that the old mathematical education is only the calculations, but now new style of mathematical education is the creativity of producing mathematical theorems.

Doing mathematics has changed the style of class lessons. The role of teacher is not teaching the mathematics to student. The role of teacher is to help the student think and make the theorem.

The Role of Teacher

It is known that the teacher's role is to transfer knowledge to students. In schools a great deal of knowledge is taught to students. The history of the human race is very long, and out of this history a great pool of knowledge has been created. As a result, students must learn many things. The teacher aims to teach as much as possible to his/her students at school. It is very important that this pool of knowledge is transferred to the students by the teacher. The teacher believes in the efficiency of the education system. The teacher gives some knowledge to the students. This flow of knowledge is from teacher to student, and not the other way around. There is not a flow from the student to the teachers. There is no change in this flow of knowledge. The teacher always passes on knowledge to students, and the students acquire it from the teacher only. We now hold doubts about this flow of knowledge. It is doubtful that the role of the teacher is to transfer knowledge to student. This fixed inflexibility is not good. It is desirable that the student relearns a great deal knowledge with the teacher. The work of the teacher is not to give knowledge to student. If his /her work entails learning with the student, the role of the teacher must change in order to help the student. A flexible approach to teaching is the style of future. At present, students dislike

mathematics as a subject in school. We wish to change the current method of education. Now we have developed a new technology which will be a benefit to education in school and home. For the students the educational helper is both the teacher and the new technology. In the information-oriented society this technology is useful for education. The student learns everything by using new technology in the school. The teacher must not be left with no work leaving their work in the hands of technology. The role of the teacher must change from only teaching to helping the students' learning process. The role of the teacher is to help to students. Both the teacher and the students, together, are the learners. Our aim is for knowledge to be transferred to the student from the teacher by using technology. It is important for the students that education is relearning by using this new technology, not teaching only and what is given to them by the teacher. However, the role of the teacher is more important to the students' learning with technology. We want the students' experience in school to be an interesting and rewarding one. So using technology in school we think that the role of teacher is changed to that of learning from that of teaching. It is important to make this new role for the teacher using technology. The system of the school changes in every part. The role of teacher is not giving knowledge to students. It is creating ideas for the students who have many possibilities.

We will show the new style of lessons with new technology. We can understand the role of teacher in the lessons. Our mathematical education is based on the method of problem solving. The teacher does not teach any new material in the lessons. The students try to create the solution of mathematical problems. So in this lesson the student is the leading actor.

We divide the teaching method with three steps,
    Step 1: Group activity
    Step 2: Presentation/discussion with reports
    Step 3: Class solving problems together

Technology can be used in every subject. The student and the teacher can learn every subject with technology. Education involves both the teacher and the student. The classroom is not silent all of the time during the lesson. The teacher waits for a long time. If the student can not understand the meaning of the problem, the teacher helps the student. We think that it is very difficult for the teacher to wait during the lessons. The role of teacher is waiting, not teaching the lesson. For the students it is important to explore and to be creative with knowledge. We think that this is the new role created by the new technology and the student.

Changes in Classroom Teaching

We think that there are a number of styles for education in the world. In our culture, the educational form is difficult. When I went to the conference at USA, there is no desk in this conference room. There were not facilities to write notes. But there is no one who was writing anything on their notebooks. American style and Japanese are different. We show the three styles.

    (1) the Visual type - Japanese Case
    (2) the Auditory type - American Case
    (3) the Body type - Using New Technology Case

For the American style is important to use the sense of hearing. So their class lessons are based on dialogue. The teacher plans to talk every time. The students do not write in their notebooks, they

want to hear everything. So the teacher uses the OHP (overhead projector). They do not use the desk. This is the Auditory style. In Japanese case, the Visual type uses writing. The blackboard is a useful tool for teacher and the student.   Student wants to write in their notebooks, and the teacher also writes on the blackboard. In the Japanese style the OHP is not useful, it is not easy to write on them. So the classroom is very silent, with no one is talking. The two types- the Visual type and the Auditory type- are representative types.

Now we show the new type of the education with technology. This type uses every sense. What is the body type with technology? What does the teacher teach in mathematics with technology? What is the use of every sense in the lesson? You have many questions such as this about the case of the body type. The student is teaching in the classroom. We want the student to learn by going by out door. They learn every thing by using fieldwork. At present the student use textbooks. They get the much knowledge from books. The body type lessons are all senses - visual sense, auditory sense and also others. We want students to get knowledge from nature and society. We have laboratories in the school. It is the most important place to study every subject. We want to build up the mathematical laboratory. The students use this laboratory and make new mathematical theorems and more. We think that the present system of school lessons has a big wall. This wall separates class rooms, and other walls divided the parts of subjects into mathematics, science, chemistry and many other parts. We want to break down these walls in the school. So the new style of lessons is using the body sense. We can use every sense in the lesson.

New Method for Teaching Mathematics

We will show the role of the teacher and the style of teaching. With technology we have a new style class room. We expect that the students will enjoy the mathematical teaching, and create new theorems in their lesson. Mathematical thinking is very difficult and we focus on the creativity necessary to make small theorems. In the past we gave mathematical knowledge to the student. But now we change mathematical education using the new technology. The teacher does not teach any new material in the classroom. The student tries to discover mathematical knowledge with the aid of technology. It has been the case that many students disliked mathematics, when they left school. The teacher does not simply teach knowledge but he/she helps the student to discover. The students have a natural mathematical creativity that the teacher can not discover.   The student creates interesting theorems in the classroom. We did not know that students have such mathematical creativity. The teacher does not teach a theorem, the students find theorems. And for the students the creating of mathematical knowledge is interesting. The students are actively involved in the creating of theorems. The students have a good time, and gain confidence in mathematics.

With the new technology for mathematics lessons the result to change the role of the teacher and the method of mathematical education. We have a new teaching style. This new method is to create and discover knowledge, not teach it. In the lessons the students come to learn that mathematics not mathematical knowledge is learned with technology. Before we thought that the student did not have ability in mathematics, and could not create new theorems in the classroom. To provide knowledge is easy for the teacher, because the teacher has a lot of knowledge. The students are only given knowledge that is unknown to them. This change in the mathematics lessons is good for both the student and the teacher. We think everyone likes mathematics learned in this way. If the teacher does not teach the mathematics to the student, then the student learns more and more and gains more knowledge.

With technology mathematical textbooks change their writing style. We have the flow from the axiomatic matter to the test in all current mathematical textbooks. It is very efficient to teach mathematics in this way. The well known theorems are covered by this method (the flow on the left of the table). This flow is thought without using technology. This is the current flow in mathematical education. Now we can use the new technology. So the style of the writing must also change. The main emphasis of the new style is exploring and making hypotheses from data. The mode of mathematical education has changed from teaching by the teacher to exploring by the student. The new flow (the flow on the right of the table) is for studying mathematics and other subjects while using technology.

<div align="center">

we are using this flow   we will have the flow with technology

axiom   problem
↓   ↓
definition   explore
↓   ↓
theorem   hypothesis
↓   ↓
proof   proof
↓   ↓
example   theorem
↓   ↓
exercise   find other problem
↓
test
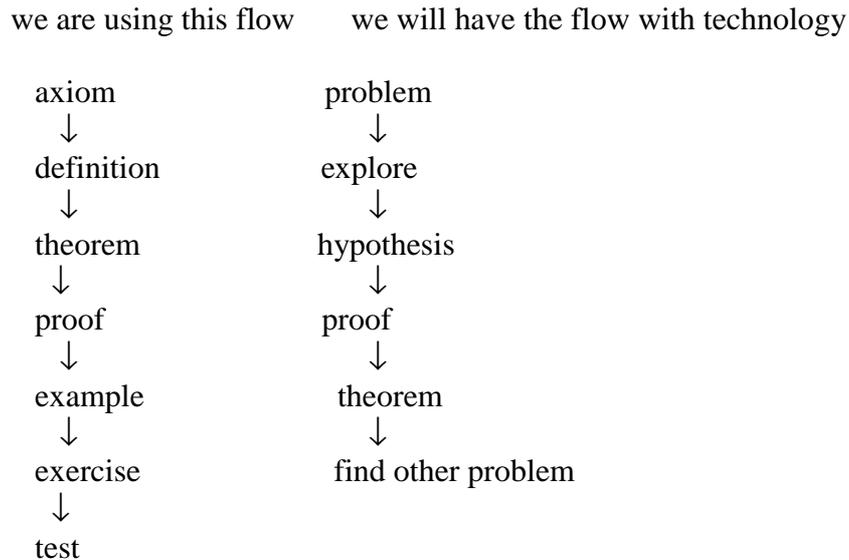
Table   Diagram of the flow of the method

</div>

Examples of creativity mathematics with TI-89

(1) Pascal's Triangle and its extensions

Pascal's Triangle was used for expanding $(x+y)^n$. But now we have a good technological tool for doing the expansion, the TI-89. If we want to calculate $(x+y)^n$, then we do not use the Pascal's Triangle.

```
(x + y)⁰ = 1                                                    1
(x + y)¹ = 1x + 1y                                           1    1
(x + y)² = 1x² + 2xy +1y²                                  1    2    1
(x + y)³ = 1x³ + 3x²y + 3xy² + y³                        1    3    3    1
(x + y)⁴                                                 1    4    6    4    1
     The Expansion of (x + y)ⁿ                          Pascal's Triangle
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$(x+y)^0 = 1$
$(x+y)^1 = 1x + 1y$
$(x+y)^2 = 1x^2 + 2xy + 1y^2$
$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + y^3$
$(x+y)^4$
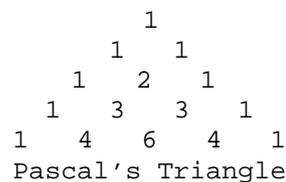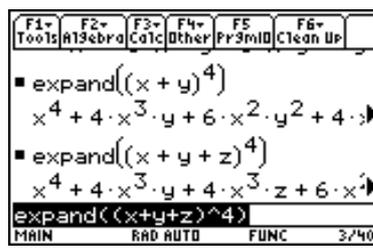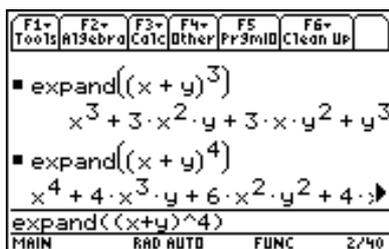
The tool ca〔 〕 ew Pascal's Triangle for

the expansion of $(x+y+z)^n$. We do not use the Pascal's Triangle as same as before. One extension is $(x+y)^n$ to the new Pascal's Triangle $(x+y+z)^n$. The standard Pascal's Triangle is one dimensional type in the plane and the new one is two dimension in the space.

```
(x + y + z)⁰ = 1
(x + y + z)¹ = 1x + 1y + 1z
(x + y + z)² = 1x² + 2xy + 2xz + 1y² + 2yz + z²
(x + y + z)³ = 1x³ + 3x²y + 3x²z + 3xy² + 6xyz + 3xz² + 1y² + 3yz² + z³
```



```
                                                  1


          1                                 2          2

                                        1       2       1
        1           1
(x + y + z)¹ = 1x + 1y + 1z    (x + y + z)² = 1x² + 2xy + 2xz + 1y² + 2yz + z²
```
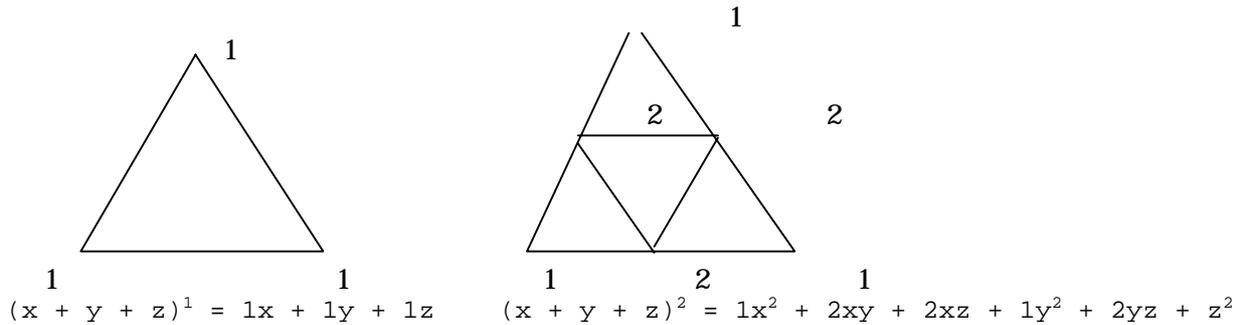
Next the figure of $(x+y+z)^3$ is the equilateral triangle and parallel lines. More interesting is that $(t+x+y+z)^n$ is three dimensional in the four dimensional space and $(t+x+y+z+w)^n$ 5-dimensional in 6-dimensional space.

Another extension is adding n numbers from the previous row. Starting with the first row of the triangle

$$\cdots 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \cdots$$

In Pascal's Triangle we add two numbers.

$$0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0$$
$$0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0$$
$$0 \quad 0 \quad 1 \quad 2 \quad 1 \quad 0 \quad 0$$
$$0 \quad 1 \quad 3 \quad 3 \quad 1 \quad 0$$

Expansion of $(x + 1)^3 = 1x^3 + 3x^2 + 3x + 1$

Next we add three numbers from the previous row

$$0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0$$
$$0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0$$
$$0 \quad 0 \quad 1 \quad 2 \quad 3 \quad 2 \quad 2 \quad 1 \quad 0$$
$$0 \quad 1 \quad 3 \quad 6 \quad 7 \quad 6 \quad 3 \quad 1 \quad 0$$

Expansion of $(x^2 + x + 1)^3 = 1x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$

The mathematical thinking is very beautiful. And the rule is found by student. This act is creativity in doing mathematics..

(2) Curvature function of y=f(x)

We use the Text in TI-89. We can get the curvature function for any function.

```
F1┬  F2┬   F3┬  F4    F5
Tools Command View Execute Find...
C:Define f(x)=tan(x)
 :f(x)→y1(x)
C:d(f(x),x)→dx
C:d(f(x),x,2)→ddx
C:ddx/(1+dx^2)^(3/2)
C:ans(1)→y2(x)
C:d(y2(x),x)→g(x)
C:solve(g(x)=0,x)→a
C:y2(x)|a→b
MAIN        RAD AUTO      FUNC
```
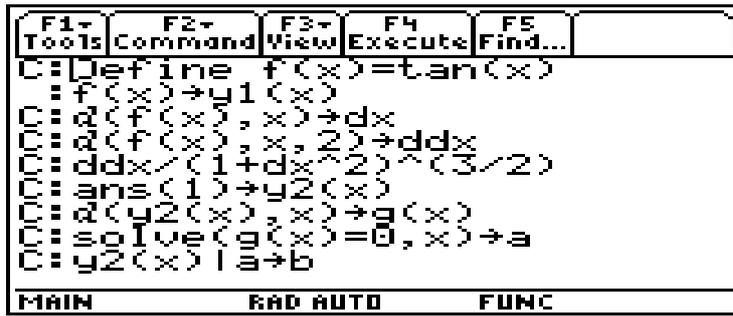
We can see the curvature function and the curve of given function y=f(x). It is very important to see the graph. The strength of the tool is for making mathematical theorems. At the 3rd conference I discussed the curvature of y=x^n. After that conference, my students came up with some theorems with the help of theTI-89. These are the relation between the curvature and the tangent line.

(1) The curvature function for y = sin(nx) (n=1,2, . . .) is drawn as the solid curve in the figure below. The maximum point of this function is at $(\frac{(2n-1)\pi}{2n},1)$ and the value of the curvature is n^2. At $x = \frac{(2n-1)\pi}{2n}$, the gradient of the tangent line of y = sin(nx) is zero.

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F1┬  F2┬  F3   F4     F5┬  F6┬ F7┬
Tools Zoom Trace ReGraph Math Draw Pen
MAIN        RAD AUTO      FUNC
```
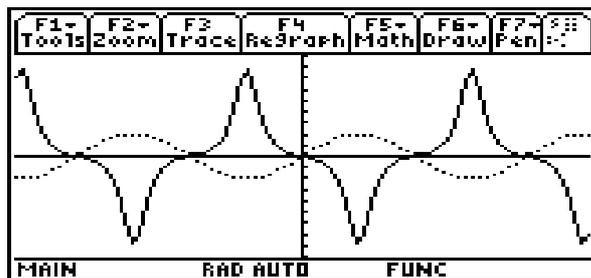
y = sin(nx) (dotted line) and the curvature function (solid line)

(2) The curvature of the function $y = e^{nx}$ is drawn as the solid line in the figure below/ The maximum point of this function is $(\frac{-\ln(2n^2)}{2n},\frac{1}{n^3})$. The value of the curvature is $\frac{2n\sqrt{3}}{9}$. At $x = \frac{-\ln(2n^2)}{2n}$, the gradient of the tangent line to $y = e^{nx}$ is $\frac{1}{\sqrt{2}}$.

```
F1┬  F2┬  F3   F4     F5┬  F6┬ F7┬
Tools Zoom Trace ReGraph Math Draw Pen
y=.708627x+.577068
MAIN        RAD AUTO      FUNC
```
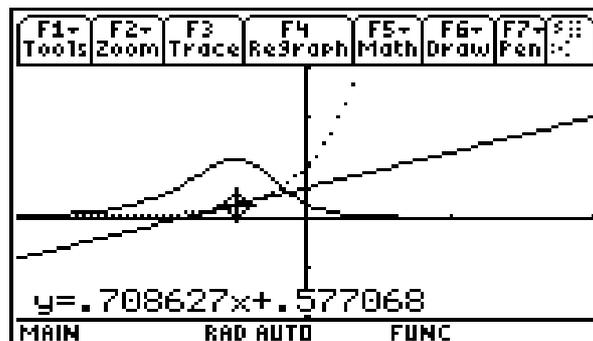
The exponential function and its curvature function

(3) The curvature function for $y = \dfrac{n}{x}$ $(n = 1, 2, \cdots)$. the maximum point of the function is at $(\sqrt{n}, \sqrt{n})$ and the value of the curvature is: $\dfrac{\sqrt{2n}}{2n}$. At $x = \sqrt{n}$, the gradient of the tangent line to $y = \dfrac{n}{x}$ is -1.
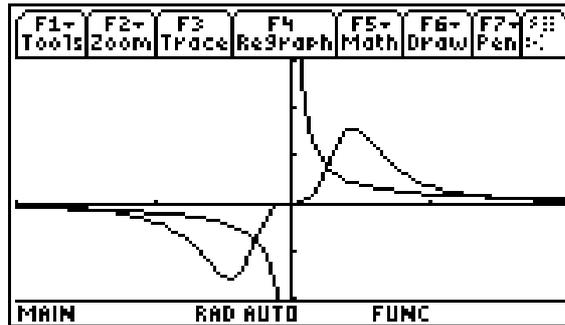
The fractional function and its curvature function

Enjoyment and Technology in the Mathematics

We show some changing styles for education with technology. The main idea is the role of teacher, who needs to use many senses and new textbooks. The larger change is that of the students' attitudes towards learning. They enjoy their mathematics lessons. Without technology they are given the mathematical theorems by their teachers and text books. But now they study mathematics by exploring and creating. This method is called the problem solving with technology. It is important to study mathematics with enjoyment. Technology has changed the attitude of students. We plan to use technology in future school lessons.

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