Contents

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Methods for the Millenium Solving Equations

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Abstract

"At school there are set approaches to solving linear equations, quadratic equations etc. At tertiary level the same thing happens except they are more difficult. One section depends on another so that it is virtually impossible to proceed to more advanced equations without being able to solve the earlier simple equations In the next millenium many students will need the results of equations without having to solve them by traditional methods. Fortunately if we adopt new curriculum to include graphical and logical computer packages in just the same way that calculators were incorporated some years ago then our new millenium will herald a new era n mathematics".

My talk will highlight how doing away with the grind of solving equations using computer packages will release the student to spend more time on formulating interesting questions and understanding the results. It is then easy to vary the data and establish any patterns in the results almost immediately. For those who are interested there will be an additional workshop session where participants can use computers equipped with Derive and G.C. to solve equations logically and graphically and then to interpret their results. A wide range of examples will be available from linear equations to differential equations.

First we will look at linear equations using "Derive" a logical computer package. The process compels you to be aware of two kinds of solution i.e. algebraic and numerical. The Algebraic solution could be a ratio for linear equations e.g. x = 9/5 or
say x = 2 for quadratic equations whereas the numerical answer would be x =1.8 or x = 1.414(3dp)

(1) 2x+x = 84 **Author equation**
Press **Solve** then **algebraic**
Press **simplify**
<u>x= 28</u>

(2) 2.07x+1.35x = 84 **Author equation**
Press **Solve** then **numerical**
Check **range 0-30**
Press **simplify**
<u>x = 24.5614</u>

(3) 2.07x-3.55 = 8.763 **Author equation**
Press **solve** then **numerical**
Check **range 0-10**

Press **simplify**
x = 5.94830

Simultaneous Linear Equations

To solve two or three simultaneous equations you author the vector option and adjust for 2 or more variables then enter equations on
separate lines. For example

(1) Solve x+3y =1 and 3x+y =11
Author vector option for 2 variables
Enter the two equations [x+3y=1,3x+y=11]
Press **OK** then press **solve** option icon,
(in the form of a magnifying glass)
then press **simplify**
Solutions[x=4,y=-1]

(2) Solve x+2y+3z = 6 3x+ y- z = -2
2x-5y+3z = 21
Author vector option for 3 variables
Enter the 3 equations
[x+2y+3z=6, 3x+y-z=-2, 2x-5y+3z=21]
Press **OK** then press **solve** option icon
then press **simplify**
Solutions[x =1,y =-2,z =3]

Extra Linear Equations

(1) 120.7S(98.7-30.3)=94.5(98.7-26.3)
Author expression,
Press **Solve numerically**
Press **simplify**
S = 0.828718

(2) 1/R = 1/150+1/300
Author expression
Press **Solve algebraically**
Change **range 0-1000**
Press **Simplify**
R = 100

(3) (5005-5000)/5005 = V/3.1010
Author expression
Press **Solve numerically**
Check **range** v **approx 107**
Use **0-108**

V = 2.99700.107

Simultaneous Equations

2/x-3/y+5/x = 4
3/x+2/y+2/z = 3
4/x+1/y-4/z =-6
Put x'= 1/x y'= 1/y z'= 1/z
Author[2x'-3y'+5z'= 4,3x'+2y'+2z'= 3,4x'+y'-4z'= -6]
Press **Solve** Option Icon
Press **simplify**
x'=-1/3 y'= 2/3 z'= 4/3
Invert x=-3 y = 3/2 z = ¾

Quadratic Equations

(1) Solve x2-3x-5=0
Author above equation
Press **Solve Algebraic**
Press **simplify**
x=3/2- (29)/2
x= (29)/2+3/2

If numerical results are needed then Press Solve Numeric only gives one result for e.g. say the negative answer -1.19258 To obtain the other answer eliminate the negative range i.e. take 0-10 only Then simplify gives answer 4.19258

(2) Now a simple problem.
Two one metre wide strips are cut from a square carpet leaving an area of 3sq.m. What is the area of the carpet?.

Let x metres be the length of a side of the square carpet. The area is (x2)sq.m. Each strip is 1*x sq.m. Therefore two strips have an area of 2x sq.m The remainder is therefore (x2-2x)sq.m. The remainder is given as 3sq.m. Therefore x2-2x = 3 Or x2-2x-3 = 0
Author
Press **Solve Algebraically**
Press **simplify**
x =-1 or x =3

The area of the carpet is 9sq.m.
Try the same problem with 3 strips and a remainder of 4 sq.m. What do you notice?
Try the same problem with 4 strips and a remainder of 5 sq.m. What do you notice? What is the answer if you cut off n strips and
the area left is (n+1)sq.m.?

Graphical Interpretation

Using the package Graphical Calculus it is possible to solve the previous problem graphically.

Using G.C. you plot 4 graphs simultaneously as in the transparency e.g.

y=x2-2x-3,y=x2-3x-4,y=x2-4x-5,y=x2-5x-6

The solutions: when y=0 are x =3, x =4, x =5, x =6

All the graphs pass through x =-1 but a negative solution is not acceptable.

Points to consider when plotting Graphs using Graphic Calculus i.e. G.C.

(1) The Scale
On G.C. this is normally x=1 y=1
This works fine for linear graphs. For quadratic functions or parabolas, it is necessary to change to x=1 y=2

(2) The Domain and Range
On G.C. the domain is set from x=-3 to x=3
In the problem we did earlier this was changed to x=-3 to x=6 . The range depended on the lowest value of the last parabola. This can be approximated by realising this occurs at x=-b/2 where b=-5 .Therefore x=5/2 A quick mental calculation gives a value of y = -12 . The range was filled in as follows y=-15 to y=10. The upper part was arbitrary but the lower part had to be lower than -12.

(3) Plotting
Before entering G.C. type the command Graphics.Com . You are then able to print screen for a full size graph or Alt. O will give a much smaller version but still quite legible.

(4) Trigonometric Graphs
The x-scale is in radians instead of degrees so the normal scale x=-3 to x=3 covers approximately -180 to 180 since a radian is approximately 60 degrees.

Tertiary Mathematics

One of the major items in tertiary mathematics is the solving of differential equations. These occur in science, engineering, medicine, biology, business and statistics. In business there is a simple economic model for the price of a product if you assume the rate of change of the price is directly proportional to the difference in the demand and the supply of the product. Let P represent the price of the product at any time t. Then if D is the demand for the product and S is the supply, the desired model is dP/dt =k(D-S)
Solving this simple 1st order equation gives P

P(t) =k(D-S)t +k

In science we have that the rate of decay of radium is proportional to the amount of radium remaining. If A = amount of radium then we have

$$dA/dt = -kA$$

which is a first order differential equation giving a solution of

A(t)= Ae-kt

Another example in statistics is the rate of population growth which is proportional to the difference between the birth rate and the death rate. If we assume that both the birth rate and death rate are proportional to the population size P then the death rate is

dP/dt = aP-bP

or

dP/dt=(a-b)P =kP

where a and b are constants associated with the birth rate and the death rate and

k =(a-b)

then we have a similar solution to the example on decay of radium with P(t) = P. I would recommend reading Rice and Strange 1994 "Ordinary Differential Equns. with Applications" 3rd Edition Brookes/Cole p.5-11 for further ex. Solving Differential Equations using Derive

A list of 5 steps will help you to load utility files which will enable you to solve both 1st order and 2nd order differential equations.

Step 1 Switch on Derive
Step 2 Press F for File
Highlight Load then utility
Step 3 Press utility
Step 4 Enter ODE 1.MTH for 1st order diff.eqns.
Step 5 Enter ODE 2.MTH for 2nd order diff.eqns.

In the case of ODE 1.MTH there are 34 files each with a different method of solving 1st order differential equations. Fortunately many are joined together under the following instructions:

If we have a first order differential equation p(x,y) + q(x,y).y' = 0 then you author

D Solve 1- Gen(p,q,x,y,c)

entering in the appropiate values of p ,and q.

If you want a specific solution then the instruction is to author

D Solve 1(p,q,x,y,xo yo)

Similarly for 2nd order equations the format is

D Solve (p,q,r)

referring to the equation y" + p(x).y'+q(x).y =r

Several examples will follow on transparencies with the graphical solutions which highlight any special values such as maximum or minimum or impossible values where the curve is discontinuous.

Solve the differential equation

dy/dx = y given that y=1 x=0

and graph the solution. In the form p(x,y)+q(x,y)y'=0

-y + 1y'=0

Therefore p=-y q=1 x =0 y =1

Author DSOLVE1(-y,1,x,y,0,1)

Solution is y = ex

Solve the differential equation

dy/dx= -2xy/1+x2 given that x=0 when y=1

Graph your solution In the form p(x,y)+q(x,y)y'=0

2xy +(1+x2)y'=0

Therefore p=2xy q=(1+x2)

Author DSOLVE1(2xy,1,1+x2,x,y,0,1)

Solution is y= 1/(1+x2)

Solve y''-3y'+2y=sin(x) given that y=0,x=0 and y'=1

Refer to y''+py'+qy=r Therefore p=-3 q=2 r=sin(x)

Author DSOLVE2_IV(-3,2,sin(x),x,0,0,1)

Solution is:

$y= 6e^x/5-3e^x/2 +3\cos(x)/10 +\sin(x)/10$

This is only a sample of the many 1st and 2nd order equations which can be solved using Derive.

Amendments have to be made to the instructions for certain 1st order equations like homogeneous, Integrating factors , Bernoulli and others. The initial conditions play an important role in 2^{nd} order equations. For example, you may know two points or one point and a gradient or possibly two gradients. To solve generally 2nd order
differential equations you need two constants, hence you need two conditions to find the values of these constants

References

Rice & Strange 1994 Ordinary Differential Equations with Applications Third Edition Brookes/Cole p.6 & p.78

Rich A. Rich R. Stoutemyr D. 1992 Derive -A Mathematical Assistant for Your Personal Computer Version 2.5

Tall D. Blokland P. & Kok Douwe 1988 A Graphic Approach to the Calculus

Extra Example Question 1

A radioactive isotope has an initial mass of 100g which decays to 75g in 2 years. If the decay equation is M = M e-Ct. Using Derive find the value of C and hence the half life of the isotope using Derive.

Using G.C. plot f(x)= 100e-0.14x. Adjust your domain to present a clearer picture. From your graph find out what mass of the isotope remains after 20 years. Also check the half life of the Isotope

Answer

M= 100g when t=0 therefore M= 100e-Ct

M= 75G when t=2 therefore 75=100e-2C
Author: 75=100e-2C
PressL C=0.14381 or approx. 0.14
Therefore M=100e-0.14tis the decay equation

The half life is the time to decay to half the mass of the isotope i.e. when M=50g
Therefore we solve 50= 100e-0.14t
Author 50= 100e-0.14t
PressL t=4.95 years

On the graph put x=20 then press enter it automatically gives y=6.08 Therefore translating this to the isotope it means that after 20 years there remains 6.08g. On the graph put x= 4.95 then press enter it automatically gives y= 50.007 which confirms the answer given by Derive.

You can access as many values of"M" ie"y" for different "t" ie "x" from a facility on the graph. You can have 8 decimal places accuracy. eg t=5 t=10 t=15 t=20 t=25 t=30 M=49.66 M=24.66 M=12.25 M=6.08 M=3.02 M=1.50

Question 2.

An object with a mass of 1kg is suspended from a spring with a spring constant of 4kg/m. The object is started from the equilibrium position with a downward velocity of 1m/sec.

Using Derive:
(a)Determine the displacement equation of the object.
(b)The amplitude & period of the oscillation
(c)Discuss the changes in the results when you quadruple the mass and when you divide the mass by 4.
(d)Illustrate these changes graphically using G.C.

Answer

If "x" represents the displacement of the object from its equilibrium position and "F" represents the corresponding restoring force of the spring, which balances the weight of the object then by Hooke's Law
F= -kx with k the spring constant,since F=mx"
then mx"= -kx gives us the second order equation to solve.

In the first case m=1 k=4 Thus x"= -4x
(a) Using Derive to solve x"=q(x) we punch in
Autonomous_Conservative(q,t,x,toxox0')
where q=-4x, x=0 when t=0 and x'=1

Autonomous_Conservative(-4x,t,x,0,0,1) gives the result x = sin2t/2

(b) Amplitude=1/2 Period=

(c) When we quadruple the mass 4x"=-4x or x"=-x

Autonomous_Conservative(-x,t,x,0,0,1) gives the result x =sint

(d) Amplitude=1 Period= 2
Similarly if m=1/4 it gives the result x =sin4x/4

Amplitude=1/4 Period= /2

These can be plotted on G.C.
The results show that if you quadruple the mass both the amplitude and period are doubled.
They also show that if the mass is divided by 4 both the amplitude and period are halved.

It actually works on the square root of the increase or decrease in mass.

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Workshop on solving equations
Using derive and G.C.
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In my talk I mentioned that you require extra instructions to cover solving various equations.

To provide for these equations a list of all instructions for both Derive and G.C. are below

Derive:

Linear: e.g. 2x+3x = 25
Author equation
Press Solve
Press Algebraic
Press Simplify
x = 5

e.g. 2x-1.5 = 3.5
Author equation
Press Solve
Press Numeric
Press Simplify
x = 2.5

Quadratic:
Author
Press Solve, Algebraic or Numeric
Remember for x- squared use x^2 Two solutions occur for Algebraic Only one occurs for Numeric, you then adjust the range to exclude this solution then try Numeric again

Trigonometric:
Author
Press Solve,Algebraic or Numeric
Press Simplify
Remember enter sine x as sin(x).
Remember normally x is in radians. Solutions will therefore be in radians
Inverse functions are Asin, Acos, Atan
For 180 type pi
e.g. Asin(x)=pi/2
x = 1

Differential Equations

1st Order:

You need utility file ODE1.MTH
The instructions are as follows
Press F for file
Highlight Load then Utility
Press utility
Highlight ODE1.MTH
Press enter
To solve generally use DSOLVE1-(p,q,x,y,c) where p and q are defined by the equation

p(x,y) +q(x,y).y =0

To solve specifically use DSOLVE1-(p,q,x,y,x0,yo). This will not cover all 1st order equations. If the programme returns the words inapplicable then you will have to identify the type of 1st order equation for example Exact_Gen(p,q,x,y,c) which simplifies to a general solution in terms of a symbolic constant Similarly for integrating factor the instruction is:-

Integrating_Factor_Gen(p,q,x,y,c)

which simplifies to a general solution in terms of a symbolic constant c. Integrating_Factor(p,q,x,y,xo,yo) simplifies
to an implicit solution Bernoulli_Gen(p,q,k,x,y,c)simplifies to a general solution of y +p(x).y =q(x).yk. Bernoulli(p,q,k,x,y,xo,yo) simplifies to an implicit solution of y +p(x).y =q(x).yk

<u>2nd Order Differential Equations</u>

As with 1st order differential equations it is necessary to load a utility file i.e. ODE2.MTH The instructions are the same except for the name. File,Load,Utility,then ODE2MTH .
To solve y"+p(x)y +q(x)y =r(x)
Author DSOLVE2(p,q,r,x,c1,c2)
Press S(Simplify)
This gives an explicit general solution in terms of arbitary constants c1 and c2 . For a specific solution there are two forms.
DSOLVE2_BV(p,q,r,x,x1,y1,x2,y2) simplifies to a specific solution that satisfies the initial conditions y = y1 at x = x1and y = y2 at x = x2 .
DSOLVE2_IV(p,q,r,x,xo,yo,vo)simplifies to a specific solution for the initial conditions y = yo and y = vo at x = xo.

<u>Graphical Work when using Derive</u>

The instructions are as follows :
Author your graph
eg y = x2 -3x +2
Press **Window** then press **2d** icon
Then press **plot**
The graph will then be plotted
To change the scale or range press

<u>Graphical Work when using Graphic Calculus (GC)</u>

The instruction "Draw graph" will plot any function eg F(x) = x2-3x+2 A normal domain is -3 to +3 . If you agree then press F1. Otherwise change domain before you enter GC enter Graphics.Com Then print screen will give a large print. Otherwise, Alt O gives a small print. The tutorial will test your ability to solve a variety of equations, graph them and draw any notable conclusions from the results.