**Fourth International Derive TI-89/92 Conference**
**Liverpool John Moores University, July 12 – 15, 2000**

# Why use CAS with the TI-89 in Mathematics Education? Classrooms experiments

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## New technology is here to stay

Television is a wonderful tool for information, education and entertainment. Some people use it sensibly and some do not. Although many people misuse television we do not forbid the television as a tool for information and education. *Similarly, we can not forbid technology in mathematics because some students will use the tool wrongly!* The new way of teaching mathematics with a computer algebra system (CAS) is gaining momentum in the world with help of the CAS TI 89. We need to start thinking how to approach mathematics with TI 89. My thoughts about it are presented here. It is very important when you start using the TI 89 that you are very well prepared and have an idea how to improve mathematics understanding with the TI 89. Let the students do their own investigations in mathematics. The questions the students will ask after their own investigations are the best way for them to learn mathematics. If students ask questions then they are really curious about your answers. This is a perfect educational situation.

## How can we create such a situation?

Perhaps giving examples from real life situations and letting them create the right questions or the right polynomials. There is some “jeopardy” in teaching mathematics this way. Here are some examples that may improve the students understanding of mathematics. I have tried these examples in my classroom with good results.

## 1. Differential Equations:

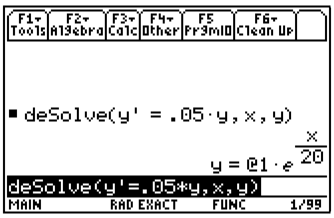
The most difficult part in differential equations’ classes is to create the equations, not solve them. That is my experience with students. I have tried to give my students some experience of differential equations applied to real life problems.

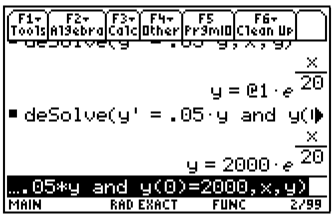
### a) The growth of money earning interest.

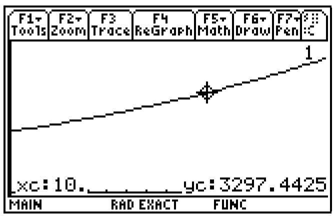
If you put 2000 Skr in the bank with an interest rate of 5% per year. The money will grow with 5% each year. The growth, if continuous, leads to a differential equation such as

$$y' = 0.05y$$

We can solve this differential equation with the TI 89. As can be seen in the screens below, we can put a starting value in and get a graph.

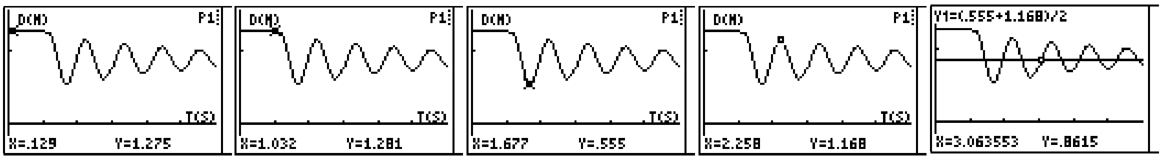


**b) Bungie Jump**

A Coca-Cola tin filled with water tied to an ordinary rubber band will be the model of a Bungie Jump. By releasing the tin over a Calculator Based Ranger placed underneath, we can sample the movement with the TI 89. The data we receive can be analysed and a lot of calculations can be performed.




The tin's mass is 0.21 kg . The rubber bands natural length (at rest) is 0.18m.

Here we can let the students take the data as homework and ask them to calculate the k-value (Young's modulus) of the rubber band, maximum speed, maximum acceleration, period and the length of time it took the tin to reach it's maximum extension (turning point).

The students either solve it with the graph or you brief the students on the formula shown below

k-value: $mgh = \frac{1}{2} \cdot k \cdot (y_{\max} - 0.18)^2$ gives k= 7.9 N/m

maximum speed from : $\frac{1}{2} k \cdot y_{jämnvikt}^2 + \frac{1}{2} mv^2 = mg(y_{jämnvikt} + 0.18)$

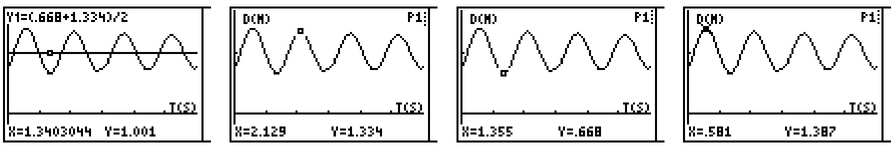
maximum acceleration from: $k \cdot (y_{max} - 0{,}18) = m \cdot a_{max}$

Period: Sine curve is $y = 0.86 \cdot \sin(\frac{2\pi}{T} \cdot t + \alpha) = 0.86 \cdot \sin(\sqrt{\frac{k}{m}} \cdot t + \alpha)$

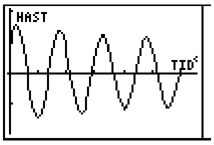
Time to the turning point is: $\sqrt{\frac{2 \cdot y_{jämnvikt}}{g}} + \frac{T}{2}$ = 0.41 s + 0.51 s =0.92 s

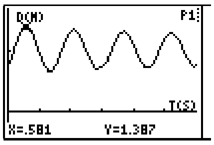
**c) The Simple Pendulum** $y'' = ay$

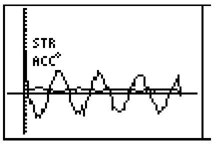
Suspend a length of string from the ceiling with a weight at the bottom and slightly displace the weight. We can get the CBR to measure the distance from the floor to the weight, we see from the data collected that simple harmonic motion.

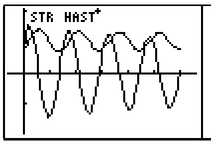



Given the displacement data, the TI 89 can numerically calculate the derivative at each step interval, graphing this gives the velocity time graph. Similarly, we can obtain the acceleration time graph. If we compare these graphs we can decide when we have the maximum speed and when we have the maximum acceleration. We have the maximum speed when we pass through the starting point and the maximum acceleration at the turning points.









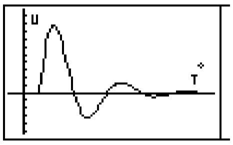

The students can use this graph as a homework to find the values of A, B, C and D $y = A\sin(Bx + C) + D$

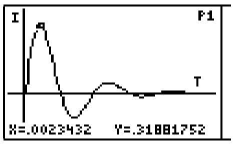
**d) Current from a circuit with resistance, condenser and coil coupled in series**

$y'' = ay' + by$

A condenser is charged and then the coil and the resistor are switched on. The voltage will now swing between the coil and the condenser. With a Calculator Based Laboratory we can measure the voltage, look at the graph of it. My value was R=25 ohms, C=22µF and L=0.089 H. The voltage was 27V.

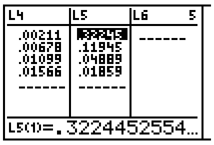
The differential equation of this voltage “swing” is $y'' + \frac{R}{L}y' + \frac{1}{LC}y = 0$

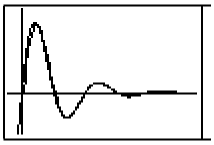
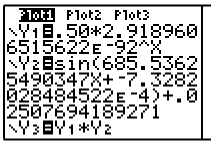


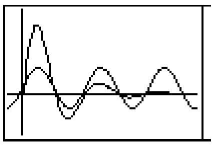
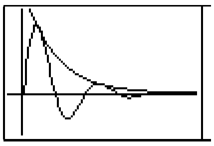
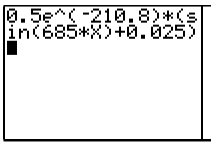



If we take the absolute value of each turning point and find the exponential regression of this data will result in an exponential curve. We compute the sine regression of the curve and then we get a sine function. The amplitude (the value before sine) is set to 1. We multiply these two curves and get the result. This will be the result of the differential equation as well.

In our case, we have $\frac{R}{L} = 210.8 och \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 685$

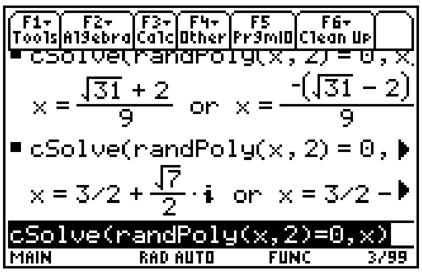


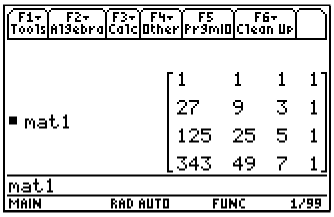





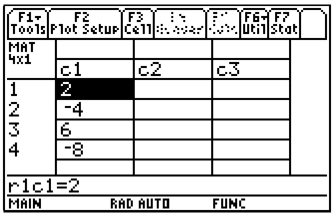

## 2. Polynomial: Which polynomial fits some given points?

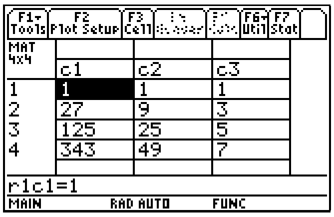
a) We randomly create a quadratic function, set it equal to zero, and solve it in one step. You get two solutions (zeros). Look at the window. Ask your students if they can suggest one or two functions that have these zeros. This is a way to show the students that every quadratic function can be written in the form $A(x - x_1)(x - x_2)$.

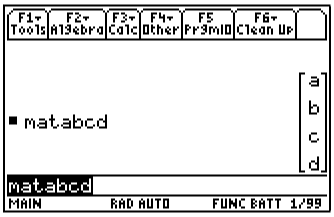


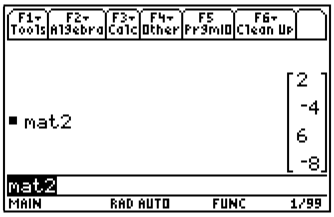
b) You have four points and you want a polynomial function to fit them. The students have to decide which order of polynomial they need and then set up an equation system. If we take the points (1,2) ; (3,-4) : (5,6) :and (7,-8). Look at the windows to follow the solution. $y = Ax^3 + Bx^2 + Cx + D$

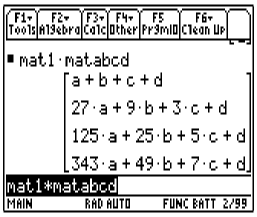


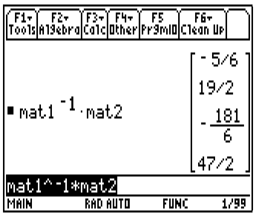


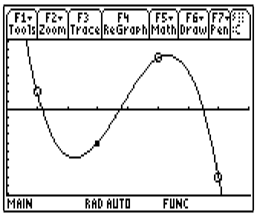




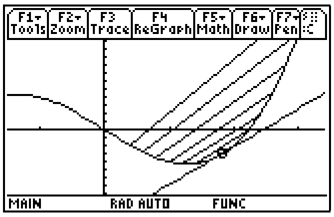




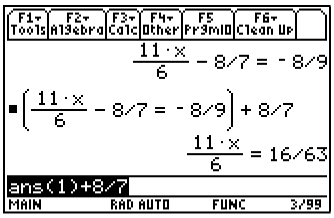

We found the function $f(x) = -\frac{5}{6}x^3 + \frac{19}{2}x^2 - \frac{181}{8}x + \frac{47}{2}$.

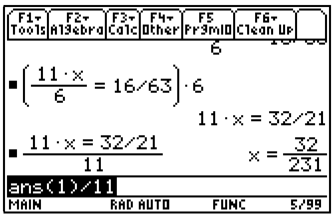
## 3. Animation of the definition of the derivative!

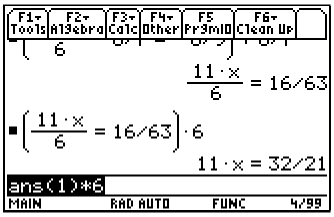


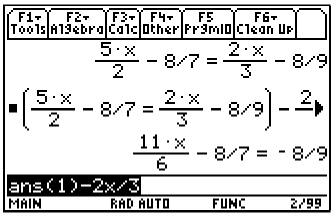

## 4. How to solve linear equations. How to solve a system of equations.

Bernard Kutzler's method to solve linear equations. The students have to think, "How will I simplify this equation?"



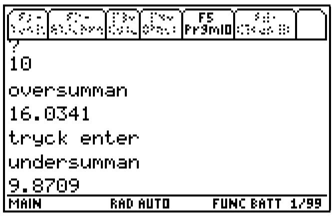


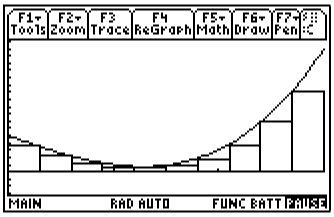


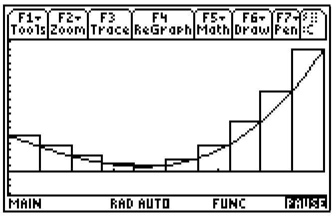



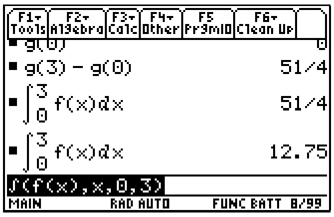

## 5. How to find an upper sum and a lower sum for a given function. (Integral.)

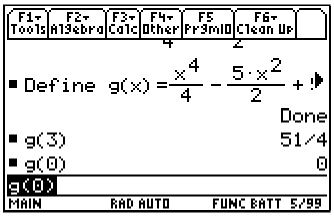
I use the following as a demonstration for upper sums and lower sums. If you take more and more steps you will get better and better value of the area below the function, i.e. integral

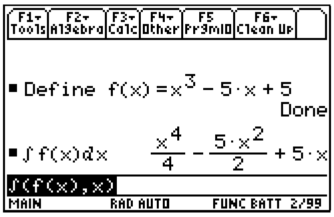


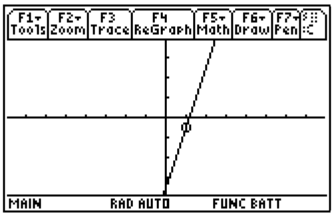



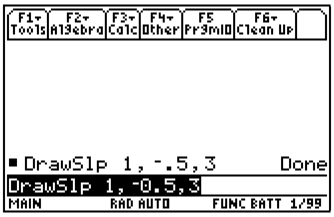



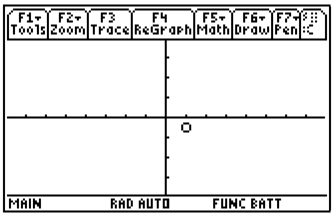



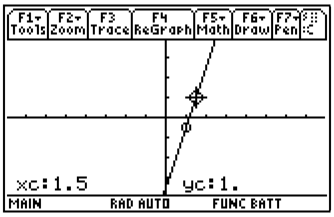



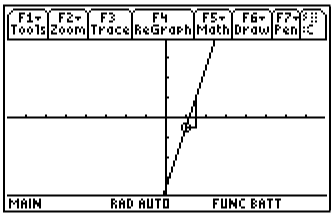

## 6. You know f ’(1)=3 and f (1) = -0.5. What is the approximate value of f (1.5)?



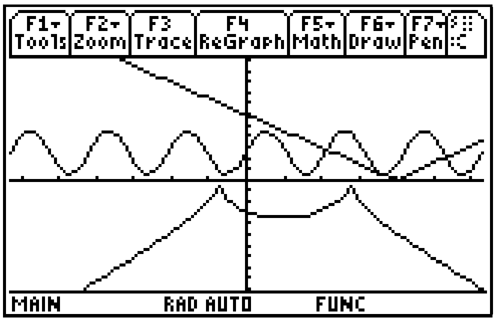




After this the students can calculate f(1,5) as (1.5-1)3=1.5 and –0.5 +1.5= 1.0

**7. Create curves such these shown in the picture.**

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