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LEARNING POWER SERIES WITH COMPUTER TOOLS

C. D'Apice*, R. Manzo and E. Zappale*****
Dipartimento di Ingegneria dell'Informazione e Matematica Applicata
Università degli Studi di Salerno, via Ponte don Melillo, 84084 Fisciano (Salerno), Italia
eEmail *dapice@diima.unisa.it , ** manzo@diima.unisa.it , *** zappale@diima.unisa.it

ABSTRACT
A key aspect of computers as aids to learning mathematics is their ability to represent mathematics in an interactive form that is more accessible to exploration by beginners than its traditional representation on paper or blackboard.

The aim of this work is to describe a package implemented in Mathematica 4.0 for the purpose of enhancing the traditional way of teaching power series through the use of interactive exercises generated randomly and the routine of step by step solution.

1. INTRODUCTION

Since their introduction, Mathematica™ and other Computer Algebra Systems (C. A. S.) have produced a great change in the approach to teaching mathematics. Students are no longer passive observers of a cognitive process, which they were forced to achieve without any possibility of interaction. On the other hand, teachers are required to present a "new" mathematics, both in contents and in pedagogy, using interactive teaching methods that are designed to increase students comprehension level. The use of new teaching tools, supported by mathematical software (therefore more advanced and somehow more stimulating) has opened new unexplored possibilities. In particular, the visual representation, realised through the C. A. S. has turned out to be a very useful tool for achieving the comprehension of abstract subjects that have no immediate geometrical interpretation.

"Teachers are using the graphing calculator in visualising concepts, explorations, experimentation, generalizing, and checking solutions to algebraic problems. In addition, training [has] led to the restructuring of topics taught in mathematics. Many teachers are omitting or giving less attention to certain topics...also, many teachers ... are using real world problems to motivate their students and they are also using the graphing utilities of the calculator ...".(Currence 1992).

Computer based education (C.B.E.) provides a unique opportunity to enlarge the learning possibilities of traditional teaching techniques. Modern theories, developed and included in computer-based learning, emphasise the importance of constructivism, of collaboration and active participation of the student in constructing mental models of mathematical knowledge. Efficient computer based tutors can be included in a Notebook of Mathematica™ 4.0. This is a complex system of numerical and symbolic analysis, and, thus, a great aid to all those people who take mathematics at every level.

The aim of this work is to describe a prototype of a package for teaching mathematics focused on power series.

Using the tools supplied by Mathematica™, it has been possible to construct routines that allow the student to visualise “a guided development” of the calculation of the radius of convergence and the sum of a given ”class” of power series.

A preliminary theoretical study phase was necessary to localise a finite number of elementary “types” into which every power series can be classified and, thus, implement the code. The package is structured in a hypertextual way, including a theoretical part and a guided laboratory for the practical session. This prototype has been realized in the University of Salerno, at the Department of Information Engineering and Applied Mathematics, which has long been interested in the study of new and effective strategies. In the tutorial the main properties of power series are described and the main theorems (differentiation, integration ...) are introduced.

The structure of the laboratory part has been planned in such a way to make the interface particularly friendly for the user. In fact, buttons directly recall the implemented functions without need of previous knowledge of the syntax. Students can choose between different types of exercises, recall the theory every time they need to see it, use the help button that explains the insertion of the arguments, and carry out by hand exercises that are posed by the system.

2. DEVELOPMENT ENVIRONMENT

We have chosen Mathematica™ 4.0 as development environment because it offers a combination of computational sophistication and programmability which makes it ideal for prototyping and developing complex applications. This software is in fact able to execute numerical and symbolic computation, and to manipulate (generate and modify) images and graphics in two and three dimensions. Mathematica™ 4.0 also allows one to easily create or modify a nice user interface building buttons and palettes, input forms and dialogs, and even fully interactive documents.

We introduced buttons and hyperlinks to allow students to see again, during the lesson, definitions and concepts that they cannot remember at the moment. We have decided to choose such tools since they allow students in difficulty an easy consultation and immediate verification.

3. STUDENTS USING THE PACKAGE

The power series package has been used as a subsidiary tool in the learning process of Engineering students. In a first step the theoretical concepts on power series have been introduced. Later, the students have been subdivided in small groups (of twenty persons each) in order to participate to laboratory practice session an hour each week. During these sessions, students have been allowed to use the package. They have been faced with two alternatives:

a) to study a power series randomly generated by the system,
b) to give the system a power series invented by themselves or taken from a book.

They have been pleased with the didactic innovation and the step by step development of the exercises turned out to be very rewarding. For the others, the complete freedom of choice for the exercises allows student to check exercises that they have had problems solving. It also helps to dissipate doubts on the understanding of elementary exercises which students would have avoided asking questions about. (In the laboratory activities the cognitive process is in the hands of students.) The feedback supplied by the system in the case of conceptual errors has also proved useful. If the student, for example, asks the system to compute the sum of divergent powers series the system notes that, in this case, the sum does not make sense.

A student dealing with power series must face three main problems: to verify convergence, to compute the radius of convergence and the sum. We need to pay attention to the calculation of the latter, which is dealt with only marginally in textbooks. At the same time we have perceived the need to give the students a complementary tool in the study of a complex subject on which they meet a lot of problems during their examinations. The application supplies an automatic development in which the system gives back the sum of the series calculated by the Mathematica™ kernel and a step by step verification with a detailed explanation of all steps and the theorems that were used. Once the series has been split the student can check the procedure with which single pieces have been computed. This allows the student to visualise the development of the parts he/she wishes to see in full detail.

The package emphasizes the possibility, in any mathematical problem, of going back to an already known one by means of simple steps. In particular, the technique adopted for the solution evidences the sum’s computation of the power series of the considered "class" can be lead back to a geometric series by means of integrations and differentiations.

4. AN EXAMPLE SESSION

In this section an example of a working session follows. Several figures are show of the lessons and the laboratory notebooks.

Figure1. In this figure we show an example of a step by step solution for computing a sum.

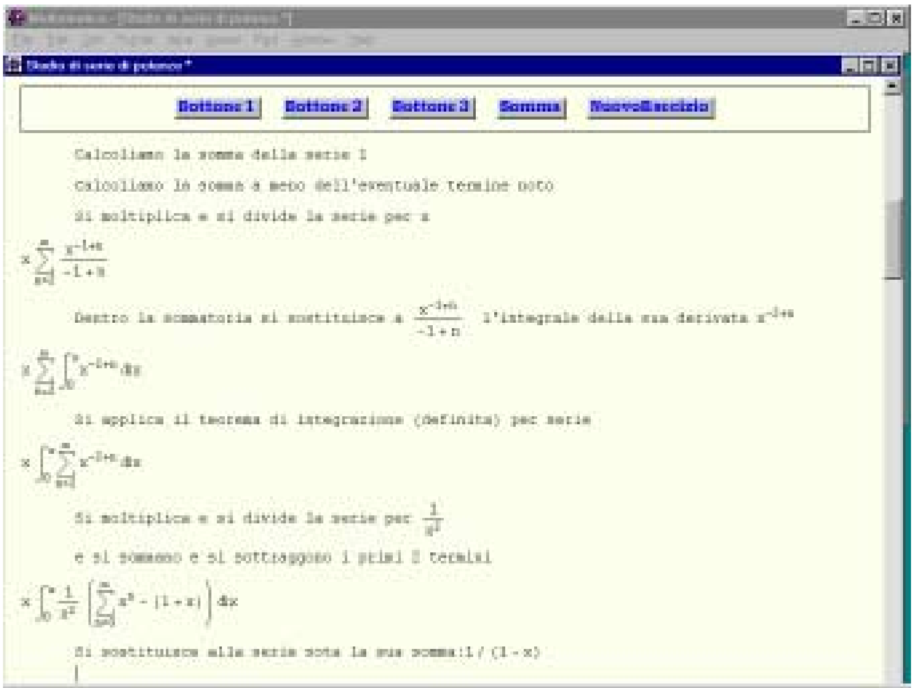

Figure 2

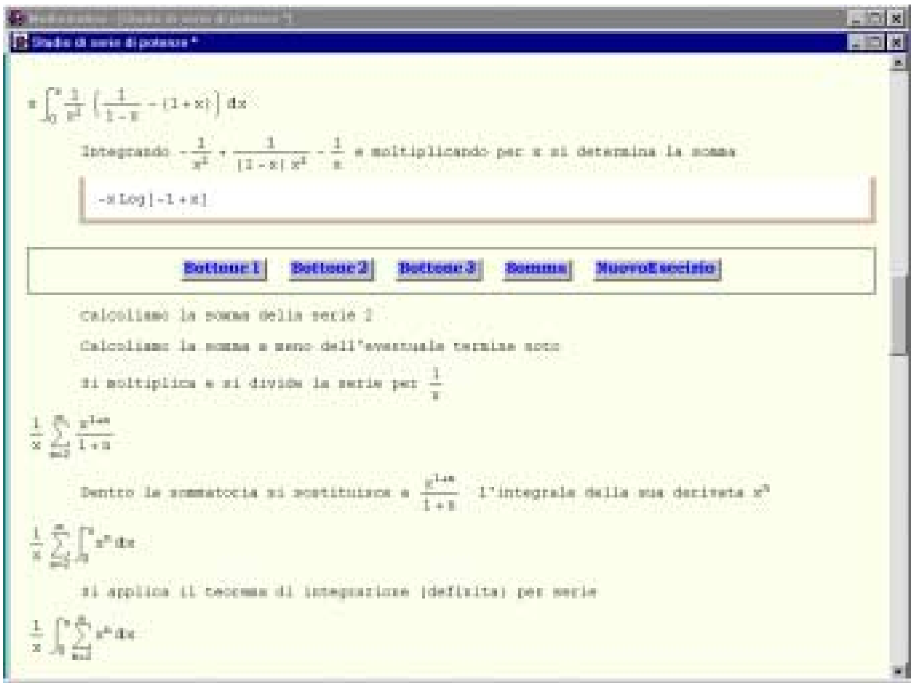

Figure 3

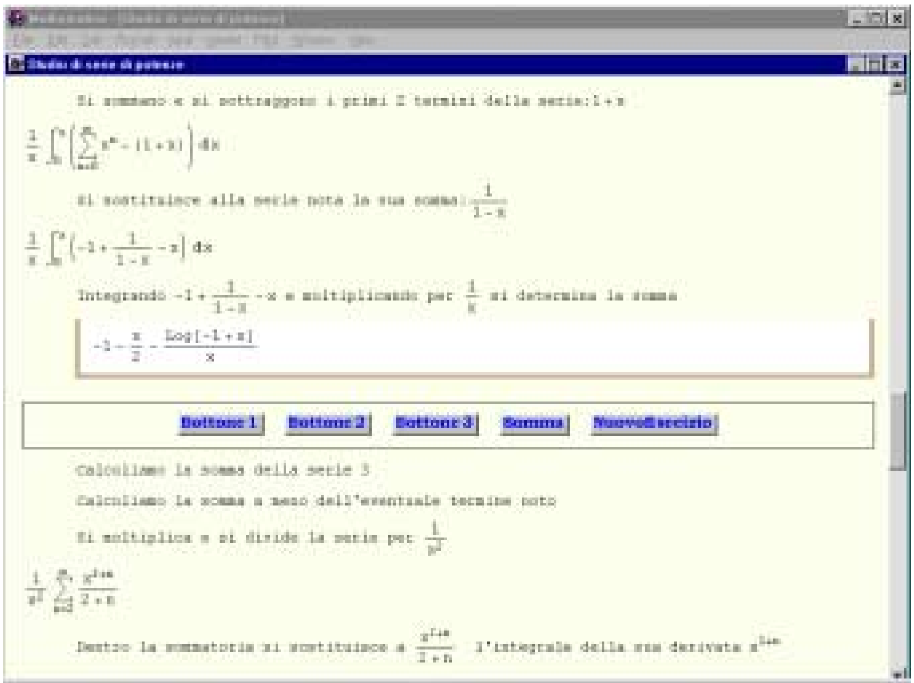

Figure 4

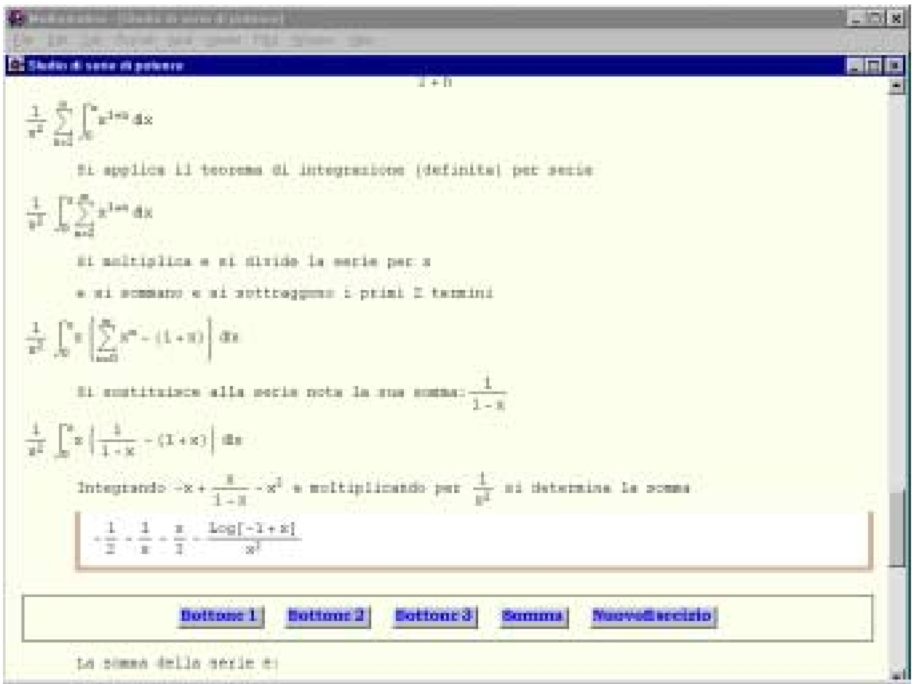

Figure 5

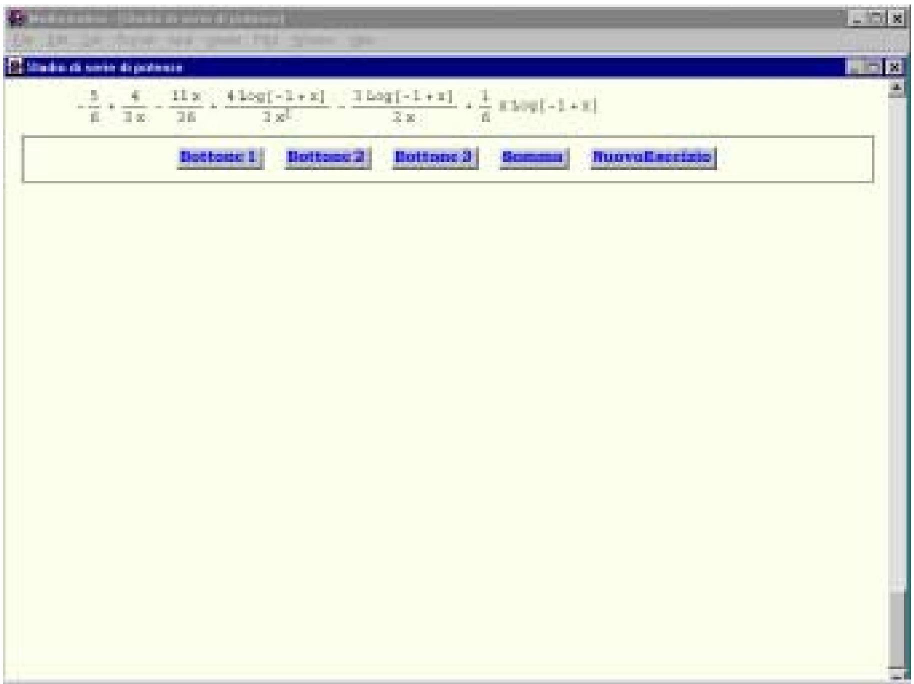

Figure 6

In Figures 2-6 we have reported the procedure with which single pieces have been computed after the series has been split.

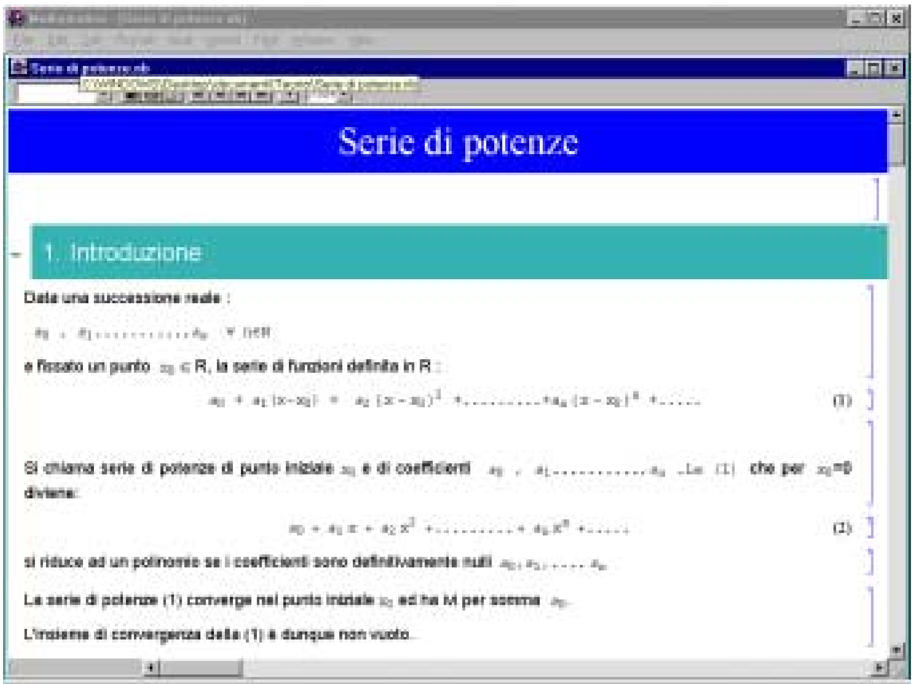

Figure 7. In this image we show the first page of the theory section.

CONCLUSIONS

From the educational point of view, the possibilities offered by Mathematica™ 4.0 are very useful for producing efficient computer based tutorials. Mathematica™ allows for the realisation of training modules having high cognitive and didactic content. They allow the possibility for verification and at the same time stimulate the search for intuitive alternatives (that do away with the outlines of routines implemented with mathematical software). The realisation of a computer-based tutor is not enough. It is necessary to measure the effects of the use of the new technology, to estimate the changes, and the improvements that it produces in the level of knowledge of the students.

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